

Factors, Fractions, and Exponents

BEFORE

In previous chapters you've...

- Multiplied and divided numbers
- Compared and ordered integers

Now

In Chapter 4 you'll study...

- Factoring numbers
- Simplifying fractions
- Multiplying and dividing expressions with exponents
- Reading and writing numbers using scientific notation

WHY?

So you can solve real-world problems about...

- geography, p. 182
- orangutans, p. 193
- computer memory, p. 199
- bubbles, p. 205



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Chapter Warm-Up Game

Review skills you need for this chapter in this quick game.

Key Skill:

Whole number division

BICYCLE MATH

HOW TO PLAY

1 PICK the number with each letter that divides evenly into the bold number in the matching fact.

- A. 4, 8, 42
B. 7, 13, 23
C. 15, 17, 21

2 USE the answer for each letter to evaluate the expression below. The value of the expression is the world record bicycle speed in miles per hour, set by Fred Rompelberg in 1995.

$$A(B + C) + 30.9$$



Algebra and Rational Numbers

Chapter 4 Factors, Fractions, and Exponents

- Find greatest common factors and least common multiples.
- Identify equivalent fractions and write fractions in simplest form.
- Use rules of exponents and scientific notation.

Chapter 5 Rational Number Operations

- Perform operations with fractions, mixed numbers, and decimals.
- Compare and convert between fractions, mixed numbers, and decimals.
- Find the mean, median, and mode(s) of a data set.

Chapter 6 Multi-Step Equations and Inequalities

- Write and solve multi-step equations and inequalities.
- Find circumferences of circles.

Chapter 7 Ratio, Proportion, and Percent

- Write and identify ratios and rates.
- Write and solve proportions.
- Use proportions and the percent equation to solve percent problems.



From Chapter 5, p. 223

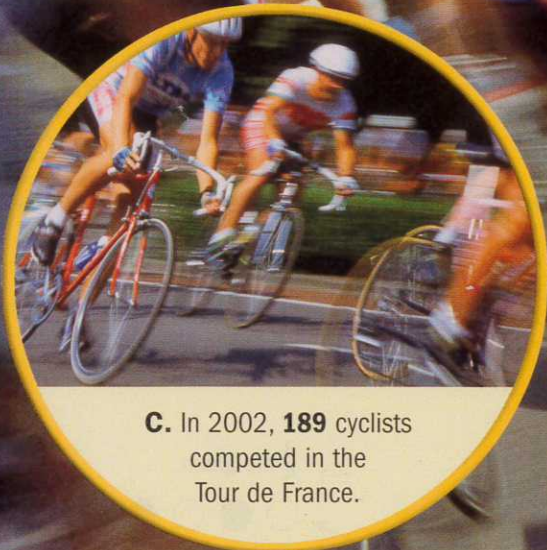
Which fruit juice blend do the most students prefer?



A. In **1884**, the “safety bicycle,” a bicycle resembling the ones we use today, was invented.



B. In 2001, **39,000,000** people in the United States rode a bicycle more than once.



C. In 2002, **189** cyclists competed in the Tour de France.

Stop and Think



- 1. Writing** A student thinks that 42 divides evenly into 1884 because 42 divides evenly into 84. Explain what is wrong with the student's reasoning.
- 2. Critical Thinking** What number will divide evenly into any even number? Explain.

CHAPTER 4

Getting Ready to Learn

Word Watch

Review Words

power, p. 20
base, p. 20
exponent, p. 20
fraction, p. 707

Review What You Need to Know

Using Vocabulary Copy and complete using a review word.

- In the expression 2^6 , 2 is called the ?.
- The expression 3^7 is a(n) ? of 3.
- In the expression 5^4 , 4 is called the ?.

Evaluate the expression. (p. 20)

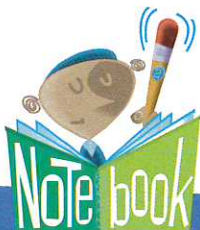
- $3^2 \cdot 3$
- $(2 + 3)^2$
- $4^2 \div 4^2$
- $(6 - 5)^7$
- $3^2 + 4 \cdot 5$
- $12 - 8 \div 2^2$
- $7^2 - 3^3$
- $6^2 \div 2^2$

Simplify the expression by combining like terms. (p. 85)

- $7x + 4 - 3x$
- $-2x + 5x - x$
- $x + 4 - 12x$
- $6x - 5 + x$

Solve the equation. Check your answer. (p. 119)

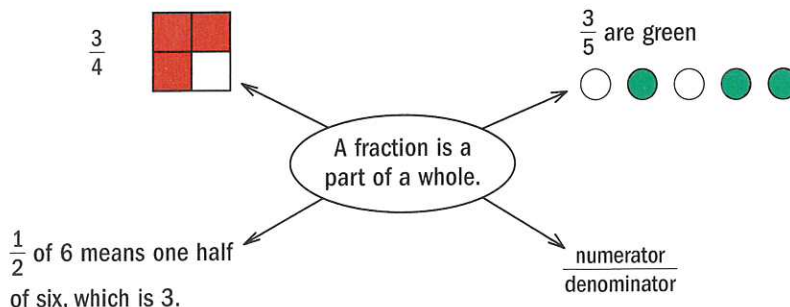
- $2a + 6 = 14$
- $8 - 4m = 20$
- $-7 + 5c = -32$



You should include material that appears on a notebook like this in your own notes.

Know How to Take Notes

Preview the Chapter Skim the content of the chapter you are about to study. If you already know something about the topic, outline what you know in your notes.



In Chapter 4, you will learn more things about fractions that you can add to your outline.

GOAL

Introduce prime and composite numbers.

MATERIALS

- paper
- colored pencils

Investigating Factors

A *prime* number is a whole number that has exactly two factors: 1 and itself. A *composite* number has more than two factors. In this activity, you will look at a number pattern attributed to Eratosthenes, a mathematician who lived in Alexandria, Egypt, around 230 B.C.

Explore

Create the Sieve of Eratosthenes with the integers 1 to 60.

- Write the whole numbers 1 to 60 in a rectangular array as shown at the right.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

- Start with the number 2. Circle it and cross out every multiple of 2 after 2.
- Move to the next number that is not crossed out, 3. Circle it and cross out every multiple of 3 after 3.
- Move to the next number that is not crossed out. Circle it and cross out all other multiples of that number.
- Repeat Step 4 until every number except 1 is either crossed out or circled.

1	②	③	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

Skip the numbers that have already been crossed out.

Your turn now

- What type of numbers are circled in your array?
- What type of numbers are crossed out in your array?

Stop and Think

- Writing** If you continued this process with the numbers 61 to 100, what type of numbers would you expect to be circled? Why?

LESSON 4.1

Factors and Prime Factorization

BEFORE

You multiplied and divided numbers.

Now

You'll write the prime factorization of numbers.

WHY?

So you can design a quilt with square patches, as in Ex. 10.

Word Watch

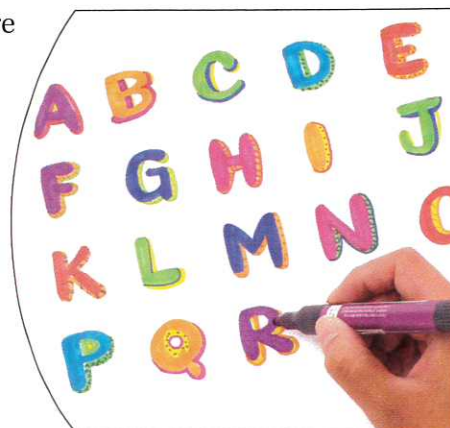
prime number, p. 169
composite number, p. 169
prime factorization, p. 169
factor tree, p. 169
monomial, p. 170

In the Real World

Lettering Members of the art club are designing their own lettering style, as part of their school arts program. Their first project is to make posters to display their new lettering style. Each poster will display the alphabet and the digits 0 through 9.

They want each row on the poster to have the same number of letters or digits. How many ways can they arrange the rectangular display?

You can use factors to determine how many arrangements are possible.



EXAMPLE 1 Writing Factors

Each arrangement will contain a total of 36 letters and digits (26 letters and the digits 0 through 9). To find the number of possible rectangular arrangements, first find the factors of 36.

- ① Write 36 as a product of two numbers in all possible ways.

$$1 \times 36 \quad 2 \times 18 \quad 3 \times 12 \quad 4 \times 9 \quad 6 \times 6$$

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

- ② Using these factors, find all the possible rectangular arrangements.

$$1 \times 36 \quad 2 \times 18 \quad 3 \times 12 \quad 4 \times 9 \quad 6 \times 6$$

$$36 \times 1 \quad 18 \times 2 \quad 12 \times 3 \quad 9 \times 4$$

ANSWER There are nine possible rectangular arrangements.

Your turn now Write all the factors of the number.

1. 20

2. 29

3. 42

4. 57

A **prime number** is a whole number greater than 1 whose only positive factors are 1 and itself. A **composite number** is a whole number greater than 1 that has positive factors other than 1 and itself.

HELP with Review

You can use divisibility tests to help find all the factors of a composite number. For help with divisibility tests, see p. 706.

EXAMPLE 2 Identifying Prime and Composite Numbers

Write all the factors of the number and tell whether it is *prime* or *composite*.

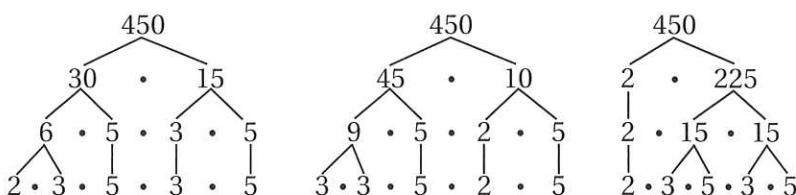
Number	Factors	Prime or Composite?
a. 32	1, 2, 4, 8, 16, 32	Composite
b. 39	1, 3, 13, 39	Composite
c. 43	1, 43	Prime
d. 76	1, 2, 4, 19, 38, 76	Composite
e. 149	1, 149	Prime
f. 189	1, 3, 7, 9, 21, 27, 63, 189	Composite

Prime Factorization When you write a number as the product of prime numbers, you are writing its **prime factorization**. One way to write the prime factorization of a number is to use a **factor tree**.

EXAMPLE 3 Writing Prime Factorization

Write the prime factorization of 450.

Three factor trees are shown. Notice that each factor tree produces the same prime factorization, differing only in the order of the factors.



So, $450 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5$.

ANSWER Using exponents, the prime factorization of 450 is $2 \cdot 3^2 \cdot 5^2$.

Your turn now Tell whether the number is *prime* or *composite*. If it is composite, write its prime factorization using exponents.

5. 24

6. 51

7. 73

8. 560

Factoring Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables. To factor a monomial means to write the monomial as a product of its factors.

EXAMPLE 4 Factoring a Monomial

Factor the monomial $12x^2y$.

Solution

$$12x^2y = 2 \cdot 2 \cdot 3 \cdot x^2 \cdot y$$

Factor 12.

$$= 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y$$

Write x^2 as $x \cdot x$.

Your turn now Factor the monomial.

9. $3mn$

10. $18t^2$

11. $14x^2y^3$

12. $54w^3z^4$

4.1

Exercises

More Practice, p. 730

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Getting Ready to Practice

1. **Vocabulary** Copy and complete: The only positive factors of a ? number are the number itself and one.

Write all the factors of the number.

2. 18

3. 27

4. 41

5. 66

Write the prime factorization of the number.

6. 28

7. 55

8. 82

9. 96

10. **Guided Problem Solving** You are making a quilt out of 120 square patches. What are the two most reasonable rectangular arrangements of the patches for the quilt?

① List all of the factors of 120.

② List all the pairs of factors from Step 1 that have a product of 120. These are the possible arrangements of the patches.

③ List the two most reasonable arrangements of the patches. Then explain why the other arrangements are not reasonable.



HELP with Homework

Example	Exercises
1	11-14, 31-38
2	15-18
3	19-25
4	27-30

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Practice and Problem Solving

Write all the factors of the number.

11. 34 12. 64 13. 108 14. 175

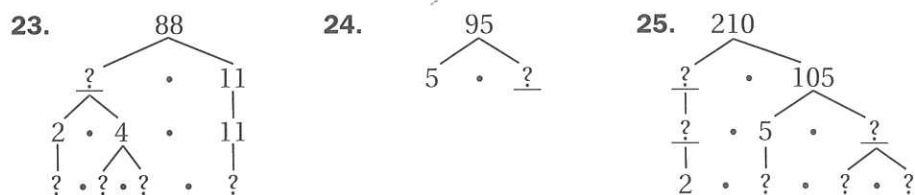
Tell whether the number is *prime* or *composite*.

15. 21 16. 45 17. 59 18. 91

Write the prime factorization of the number.

19. 56 20. 97 21. 102 22. 135

Copy and complete the factor tree. Then write the prime factorization of the number.



26. **Writing** Explain how you can create two different factor trees for 540. Do both factor trees result in the same prime factorization?

Algebra Factor the monomial.

27. $15cd$ 28. $40pq$ 29. $9a^2b^4$ 30. $48n^3m^3$

Flags of the United States Each star on the U.S. flag represents a state. Some U.S. flags had a number of stars that could have been arranged in rows and columns evenly. Describe how to do this for the number of stars indicated.

31. 15 32. 28 33. 30 34. 49

Write all the factors of the number.

35. 299 36. 336 37. 400 38. 512

Write the prime factorization of the number using exponents.

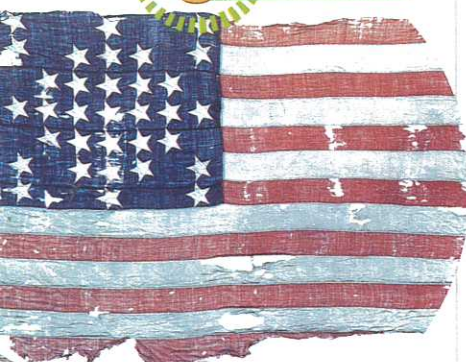
39. 280 40. 396 41. 1125 42. 2000

Critical Thinking Many even numbers can be expressed as the sum of two primes. For example, 8 can be expressed as $3 + 5$. Write the number as the sum of two primes.

43. 10 44. 16 45. 28 46. 30

What do you think?

History



Flags of the United States

The flag shown above was displayed over Fort Sumter in 1861. The flag has 33 stars. What rectangular arrangement could have been used to display the stars?



47. **Ring Toss Game** You are in charge of a ring toss game at a school fair. In this game, players try to throw a small ring over a bottle that is in a rectangular display. You have 140 bottles, and you want to arrange them in equal rows. Can you have 40 equal rows? Explain.
48. **Field Day** There are 180 students participating in field day activities. The students must be divided into teams of equal size. How many students will be on each team? List all the possibilities.
49. **Critical Thinking** Name two prime numbers that have a difference of 1.
50. **Challenge** Twin primes are prime numbers that have a difference of 2. For example, 5 and 7 are twin primes. List five pairs of twin primes other than 5 and 7.
51. **Volume** The volume of a box can be found by using the formula $\text{Volume} = \text{length} \times \text{width} \times \text{height}$. A box has a volume of 200 cubic inches. Find all possible whole number dimensions of the box.

Mixed Review

Simplify the expression by combining like terms. (Lesson 2.7)

52. $4x - 5y + 3x - 4 + y$

53. $a - 2b - 5a + 2 + 6b$

Solve the inequality. (Lesson 3.7)

54. $5x \leq -20$

55. $\frac{1}{8}y \geq 5$

56. $-9z > 108$

Choose a Strategy Use a strategy from the list to solve the following problem. Explain your choice of strategy.

57. The bookstore at your favorite mall is expanding to include the space adjacent to it. The original store was 20 feet by 25 feet. The space adjacent to it is 14 feet by 25 feet. What is the total area of the store with the additional space?

Problem Solving Strategies

- Guess, Check, and Revise
- Look for a Pattern
- Draw a Diagram

Basic Skills Find low and high estimates for the product or quotient.

58. 37×21

59. 143×58

60. $700 \div 14$

61. $1330 \div 87$

Test-Taking Practice

62. **Extended Response** The area of a rectangle is 24 square inches. Its length and width are measured in whole inches. Find all possible dimensions of the rectangle. Show your work and explain how you found all of the possibilities.

LESSON 4.2

Greatest Common Factor

BEFORE

You found the factors of a number.

Now

You'll find the greatest common factor of two or more numbers.

WHY?

So you can decide how many food baskets you can make, as in Ex. 21.



Word Watch

common factor, p. 173
greatest common factor (GCF), p. 173
relatively prime, p. 174

Activity

The lists below show the factors of the given numbers.

Number	Factors
36	1, 2, 3, 4, 6, 9, 12, 18, 36
24	1, 2, 3, 4, 6, 8, 12, 24
20	1, 2, 4, 5, 10, 20

- Which number(s) are factors of 20 and 36?
- Which number(s) are factors of 24 and 36?
- Which number(s) are factors of 20, 24, and 36?
- What is the greatest factor that is in all three lists?

HELP

with Vocabulary

The GCF is sometimes called the greatest common divisor (GCD) because it is the largest common factor that can be divided evenly into the given numbers.

A **common factor** is a whole number that is a factor of two or more nonzero whole numbers. The greatest of the common factors is the **greatest common factor (GCF)**.

One method for finding the GCF of two or more numbers is to use the prime factorization of each number. The GCF is the product of all the factors that the numbers have in common.

EXAMPLE 1

Finding the Greatest Common Factor

Find the greatest common factor of 42 and 70.

Begin by writing the prime factorization of each number. Find the product of the common prime factors.

$$42 = 2 \times 3 \times 7 \quad 70 = 2 \times 5 \times 7$$

The common prime factors are 2 and 7. The GCF of 42 and 70 is the product of these factors.

ANSWER The GCF of 42 and 70 is $2 \cdot 7$, or 14.

Two numbers are **relatively prime** if their greatest common factor is 1. For example, 8 and 15 are relatively prime.

EXAMPLE 2 Identifying Relatively Prime Numbers

Decide whether the numbers 112 and 45 are relatively prime. If they are not relatively prime, find the greatest common factor.

Begin by writing the prime factorization of each number. Then find the product of the common prime factors.

$$112 = 2^4 \cdot 7 \qquad 45 = 3^2 \cdot 5$$

There are no common prime factors. However, two numbers always have 1 as a common factor. So, the GCF is 1.

ANSWER The numbers 112 and 45 are relatively prime.

Your turn now Find the greatest common factor of the numbers.

1. 12, 18 2. 24, 60 3. 36, 90 4. 96, 120

Decide whether the numbers are relatively prime. If they are not relatively prime, find the GCF.

5. 48, 72 6. 124, 128 7. 39, 44 8. 200, 63

You can find the greatest common factor of two monomials by factoring the monomials.

EXAMPLE 3 Finding the GCF of Monomials

Find the greatest common factor of $12a^3$ and $9a^2$.

First factor each expression.

$$12a^3 = 2 \cdot 2 \cdot \color{red}{3} \cdot \color{red}{a} \cdot \color{red}{a} \cdot \color{red}{a} \qquad 9a^2 = 3 \cdot \color{red}{3} \cdot \color{red}{a} \cdot \color{red}{a}$$

The common factors are 3 and a^2 . The GCF is the product of the common factors.

ANSWER The GCF is $3a^2$.

Your turn now Find the greatest common factor of the monomials.

9. $6x$, $18x$ 10. $6xy$, $4xy^2$ 11. $15y$, $9x^2y^2$ 12. $5xy^3$, $10x^2y^2$



EXAMPLE 4 Using the Greatest Common Factor

Pep Rally Packs Students at your school are planning to hand out pep rally packs to support your school's athletic program. The students have 240 bumper stickers, 360 pennants, and 720 pencils. Every pack must have the same contents, and there should be no leftover items. What is the greatest number of pep rally packs that can be made?

Solution

You can find the greatest number of pep rally packs by finding the GCF.

$$240 = 2^4 \cdot 3 \cdot 5 \quad 360 = 2^3 \cdot 3^2 \cdot 5 \quad 720 = 2^4 \cdot 3^2 \cdot 5$$

The common prime factors are 2^3 , 3, and 5. The GCF is $2^3 \cdot 3 \cdot 5$, or 120.

ANSWER The greatest number of pep rally packs is 120. Each pack will contain 2 bumper stickers, 3 pennants, and 6 pencils.

4.2

Exercises

More Practice, p. 730



Getting Ready to Practice

- Vocabulary** Copy and complete: Six is the ? of 12 and 18.

Matching Match the pair of numbers with its GCF.

- | | | | |
|---------|----------|----------|-----------|
| 2. 6, 9 | 3. 4, 10 | 4. 5, 15 | 5. 14, 21 |
| A. 5 | B. 3 | C. 7 | D. 2 |

- Find the Error** Describe and correct the error in the solution.

$$210 = 2 \cdot 3 \cdot 5 \cdot 7$$

$$495 = 3 \cdot 3 \cdot 5 \cdot 11$$

The GCF is 5.



- Guided Problem Solving** There are 56 girls and 68 boys in a youth sports league. Each student will be placed on a team. Each team will have an equal number of players and will have the same number of girls. What is the greatest number of teams that can be formed?
 - Write the prime factorizations of the numbers 56 and 68.
 - What are the common prime factors of the two numbers?
 - Multiply the common prime factors to get the GCF. What meaning does the GCF have in the situation?



Practice and Problem Solving

HELP with Homework

Example Exercises

1. 8-11
2. 12-20
3. 22-27
4. 21, 33



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Find the greatest common factor of the numbers.

8. 3, 9, 27 9. 21, 28, 56 10. 17, 18, 20 11. 24, 36, 180

Decide whether the numbers are relatively prime. If not, find the greatest common factor.

12. 5, 18 13. 10, 25 14. 28, 42 15. 55, 72
16. 21, 66 17. 18, 216 18. 212, 312 19. 268, 515

20. **Writing** Can two even numbers be relatively prime? Explain why or why not.

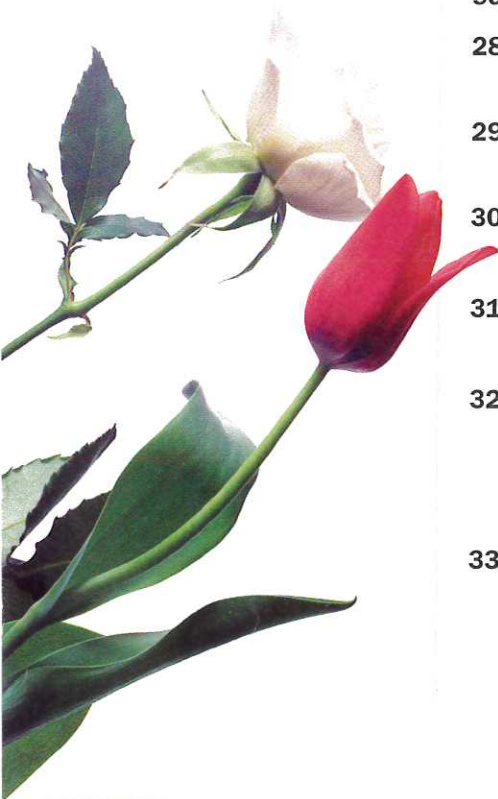
21. **Food Baskets** Your class is making Thanksgiving baskets to be distributed by a food bank. You have collected 60 cans of cranberry sauce, 120 cans of canned fruit, 90 cans of corn, and 60 boxes of muffin mixes. You want every basket to be the same with no leftover items. What is the greatest number of baskets you can assemble? What will each basket contain?

Algebra Find the GCF of the monomials.

22. $3x^2, 9x$ 23. $4z^3, 2z^2$ 24. $5t^4, 15t^5$
25. $12x^2y^2, 16xy^3$ 26. $18rs^2, 30st^2$ 27. $15bc^3, 75b^3c$

Critical Thinking Copy and complete the statement using *always*, *sometimes*, or *never*.

28. The greatest common factor of two numbers is ? equal to one of the two numbers.
29. The number 1 is ? the greatest common factor of relatively prime numbers.
30. The greatest common factor of two numbers is ? greater than both of the numbers.
31. **Critical Thinking** Name two composite numbers that are relatively prime.
32. **Carpentry** You must cut four pieces of wood that measure 36 inches, 45 inches, 108 inches, and 81 inches, into smaller, equally sized pieces. What is the longest each piece can be so that each piece is the same length?
33. **Bouquets** A florist must make a batch of identical bouquets. The florist has 360 tulips, 270 roses, and 180 lilies. There cannot be any flowers left over. What is the greatest number of bouquets that the florist can make?

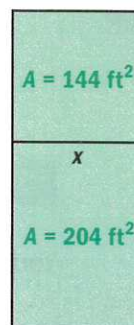




Jacob sheep can have two, four, and occasionally six horns.

34. Challenge Explain why any two prime numbers are always relatively prime. Give examples to justify your reasoning.

35. Sheep A farmer needs to build two adjacent rectangular pens for his sheep. He wants one pen to have an area of 204 square feet, and the other to have an area of 144 square feet. Fence lengths are available in one-foot increments. What is the greatest length the farmer can make the fence that is shared by the two pens?



Mixed Review

In Exercises 36–38, solve the equation. (Lesson 3.1)

36. $-6 + n = 4$

37. $n + 13 = 5$

38. $n + 2.7 = 5.7$

39. Write the prime factorization of 84. (Lesson 4.1)

Basic Skills Use a metric ruler to draw a segment with the given length.

40. 7 cm

41. 15 cm

42. 145 mm

43. 84 mm

Test-Taking Practice

44. Multiple Choice What is the greatest common factor of $4x^2$ and $6x$?

A. x

B. $2x$

C. $4x$

D. $2x^2$

45. Multiple Choice What is the greatest common factor of 144, 300, and 240?

F. 4

G. 6

H. 12

I. 60



Marble Mystery

You have a bucket full of marbles. If the marbles in the bucket are counted by twos, threes, fives, and sevens, there is exactly one left over each time. What is the fewest number of marbles that could be in the bucket?



GOAL

Use area models to find equivalent fractions.

MATERIALS

• graph paper
• colored pencils

Equivalent Fractions

You can use area models to find equivalent fractions.

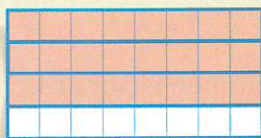
Explore

Find two fractions equivalent to $\frac{6}{8}$.

- 1 Draw a rectangle on a piece of graph paper. Divide the rectangle into 8 equal parts and shade 6 of the parts.



- 2 Look for other ways of dividing the rectangle into equal parts.



There are 4 parts
and 3 are shaded.



There are 16 parts
and 12 are shaded.

- 3 Write the equivalent fractions.

The fractions $\frac{3}{4}$ and $\frac{12}{16}$ are equivalent to $\frac{6}{8}$.

Your turn now Draw a model of the given fraction. Then find two equivalent fractions.

1. $\frac{4}{6}$

2. $\frac{10}{12}$

3. $\frac{4}{16}$

4. $\frac{10}{16}$

Stop and Think

5. **Writing** How can factoring both the numerator and denominator of a fraction help to write an equivalent fraction?

LESSON 4.3

Simplifying Fractions

BEFORE

You evaluated numerical expressions.

Now

You'll simplify fractions.

WHY?

So you can find the fractions of threatened species, as in Ex. 39.

Word Watch

simplest form, p. 179
equivalent fractions, p. 179

In the Real World

History One of the Czech Republic's royal coronation jewels is the St. Wenceslas crown. It was made around 1345 and is decorated with 44 spinels, 30 emeralds, 22 pearls, 19 sapphires, and 1 ruby. What fraction of jewels in the crown are emeralds? You will see how to solve this problem in Example 1.

Fractions A *fraction* is a number of the form $\frac{a}{b}$ ($b \neq 0$) where a is called the numerator and b is called the denominator. A fraction is in **simplest form** if its numerator and denominator have 1 as their GCF. **Equivalent fractions** represent the same number. They have the same simplest form.



EXAMPLE 1 Writing a Fraction in Simplest Form

Write the fraction of jewels in the crown that are emeralds. Then simplify.

$$\frac{\text{Number of emeralds}}{\text{Total number of jewels in the crown}} = \frac{30}{116}$$

Method 1: Write the prime factorization of each number.

$$30 = 2 \cdot 3 \cdot 5 \quad 116 = 2^2 \cdot 29$$

The GCF of 30 and 116 is 2.

$$\begin{aligned} \frac{30}{116} &= \frac{30 \div 2}{116 \div 2} && \text{Divide numerator and denominator by GCF.} \\ &= \frac{15}{58} && \text{Simplify.} \end{aligned}$$

Method 2:

$$\begin{aligned} \frac{30}{116} &= \frac{2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 29} && \text{Write prime factorizations.} \\ &= \frac{\cancel{2}^1 \cdot 3 \cdot 5}{\cancel{2}_1 \cdot 2 \cdot 29} && \text{Divide out common factor.} \\ &= \frac{15}{58} && \text{Simplify.} \end{aligned}$$

ANSWER The fraction of jewels that are emeralds is $\frac{15}{58}$.

EXAMPLE 2 Identifying Equivalent Fractions

Tell whether the fractions $\frac{3}{8}$ and $\frac{18}{48}$ are equivalent.

Write each fraction in simplest form.

$$\frac{3}{8} \text{ is in simplest form. } \frac{18}{48} = \frac{18 \div 6}{48 \div 6} = \frac{3}{8}$$

ANSWER The fractions are equivalent.

HELP with Solving

A fraction has many equivalent fractions. There are other correct answers to Example 3.

EXAMPLE 3 Writing Equivalent Fractions

Write two fractions that are equivalent to $\frac{4}{10}$.

Multiply or divide the numerator and denominator by the same nonzero number.

$$\frac{4}{10} = \frac{4 \times 3}{10 \times 3} = \frac{12}{30}$$

Multiply numerator and denominator by 3.

$$\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$$

Divide numerator and denominator by 2, a common factor of 4 and 10.

ANSWER The fractions $\frac{12}{30}$ and $\frac{2}{5}$ are equivalent to $\frac{4}{10}$.



Spinel



Sapphire

Your turn now Use the information on page 179. Write the fraction of jewels in the crown that are the given jewel. Simplify if possible.

1. pearls 2. sapphires 3. spinels

Write two fractions that are equivalent to the given fraction.

4. $\frac{8}{16}$ 5. $\frac{9}{15}$ 6. $\frac{10}{12}$ 7. $\frac{21}{24}$

You can use Method 2 of Example 1 to simplify fractions that contain variable expressions.

EXAMPLE 4 Simplifying a Variable Expression

$$\frac{14x}{7xy} = \frac{2 \cdot 7 \cdot x}{7 \cdot x \cdot y}$$

Factor numerator and denominator.

$$= \frac{2 \cdot \overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{x}}}{\overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{x}} \cdot y}$$

Divide out common factors.

$$= \frac{2}{y}$$

Simplify.

HELP with Notetaking

You may want to add information on simplifying fractions with variable expressions to the outline that you started on p. 166.

EXAMPLE 5 Evaluating a Variable Expression

Evaluate the expression $\frac{-4x^3}{2x}$ when $x = 5$.

$$\frac{-4x^3}{2x} = \frac{-1 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x}{2 \cdot x}$$

Factor numerator and denominator.

$$= \frac{-1 \cdot 2 \cdot \cancel{2}^1 \cdot \cancel{x}^1 \cdot x \cdot x}{\cancel{2}_1 \cdot \cancel{x}_1}$$

Divide out common factors.

$$= -2x^2$$

Simplify.

$$= -2(5)^2$$

Substitute 5 for x .

$$= -50$$

Evaluate powers and simplify.

Your turn now Simplify the variable expression.

8. $\frac{4xy}{6x}$

9. $\frac{32a}{8ab}$

10. $\frac{2m^3}{6m}$

11. $\frac{5r^2s}{10rs}$

12. Evaluate the expression $\frac{35a^4}{-5a^2}$ when $a = 3$.

4.3 Exercises

More Practice, p. 730



INTERNET
eWorkbook Plus
CLASSZONE.COM

Getting Ready to Practice

Vocabulary Tell whether the fractions are equivalent.

1. $\frac{2}{3}, \frac{4}{6}$

2. $\frac{15}{25}, \frac{3}{4}$

3. $\frac{15}{18}, \frac{5}{6}$

4. $\frac{21}{49}, \frac{3}{7}$

Write the fraction in simplest form.

5. $\frac{10}{15}$

6. $\frac{16}{20}$

7. $\frac{25}{40}$

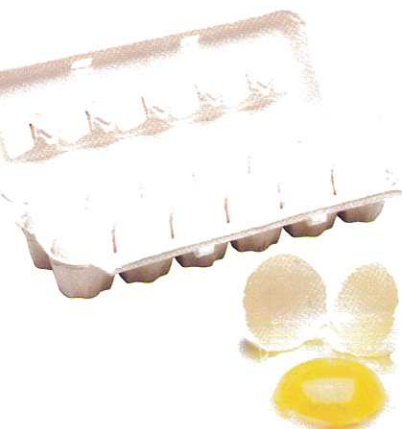
8. $\frac{36}{72}$

9. Write two fractions that are equivalent to $\frac{1}{5}$.

10. Write two fractions that are equivalent to $\frac{12}{15}$.

11. Evaluate the expression $\frac{8x^3}{2x}$ when $x = 4$.

12. **Eggs** In a carton of one dozen eggs, 2 eggs are broken. What fraction of the eggs are broken? What fraction of the eggs are unbroken? Write your answers in simplest form.



HELP with Homework

Example	Exercises
1	13-16, 33-39
2	21-24
3	25-28
4	17-20
5	29-32



- More Examples
- eTutorial Plus

Practice and Problem Solving

Write the fraction in simplest form.

13. $\frac{39}{52}$ 14. $\frac{18}{27}$ 15. $\frac{-9}{72}$ 16. $\frac{-49}{56}$
17. $\frac{4ab}{8a}$ 18. $\frac{6c}{18cd}$ 19. $\frac{-9rst}{30rs}$ 20. $\frac{25xy}{35xyz}$

Tell whether the fractions are equivalent.

21. $\frac{4}{5}, \frac{20}{25}$ 22. $\frac{21}{28}, \frac{1}{3}$ 23. $\frac{7}{35}, \frac{2}{10}$ 24. $\frac{32}{72}, \frac{4}{9}$

Write two fractions that are equivalent to the given fraction.

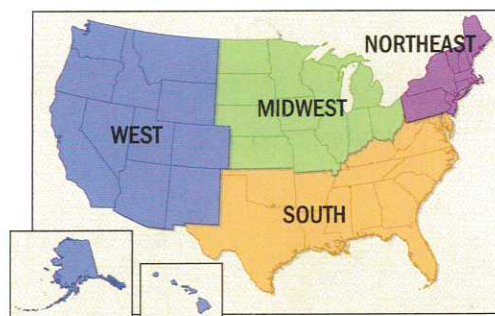
25. $\frac{45}{90}$ 26. $\frac{36}{81}$ 27. $\frac{24}{60}$ 28. $\frac{48}{140}$

Evaluate the expression when $x = 3$ and $y = 5$.

29. $\frac{3x}{x^3}$ 30. $\frac{2y^2}{-5y}$ 31. $\frac{5y}{y^2}$ 32. $\frac{4x^4}{24x^3}$

Geography Write the number of states in the region as a fraction of all of the states. Write your answer in simplest form.

33. Northeast: 9
34. Midwest: 12
35. South: 16
36. West: 13



The U.S. Census Bureau divides the 50 states into 4 regions.

Extended Problem Solving In Exercises 37–39, use the table. It gives information about animals in Peru. Write your answer in simplest form.

37. What fraction of mammal species are threatened?
38. What fraction of reptile species are threatened?

39. **Number Sense** Use rounding to compare the fractions of threatened mammal and bird species. Then write the fractions of threatened species in each group (mammals, birds, and reptiles) in order from least to greatest.

Peruvian Animal Species		
	Known	Threatened
Mammals	460	46
Birds	1541	64
Reptiles	360	9



Yellow-faced Parrot

Write the fractions in simplest form. Tell whether they are equivalent.

40. $\frac{30}{60}, \frac{27}{54}$

41. $\frac{24}{40}, \frac{30}{50}$

42. $\frac{15}{18}, \frac{36}{48}$

43. $\frac{24}{32}, \frac{15}{24}$

44. $\frac{30}{75}, \frac{75}{105}$

45. $\frac{45}{54}, \frac{90}{108}$

46. $\frac{54}{96}, \frac{144}{256}$

47. $\frac{84}{112}, \frac{168}{192}$

Write the fraction in simplest form.

48. $\frac{-2x^3y}{xy}$

49. $\frac{5xy^2z}{5xz}$

50. $\frac{2x^3y}{-3xyz}$

51. $\frac{-8^2z^2}{16x^2yz}$

52. **Critical Thinking** If you divide the numerator and denominator of a fraction by a common factor, will the resulting fraction always be in simplest form? Give an example to justify your answer.

Tell whether the fractions are equivalent.

53. $\frac{4abc}{5ab}, \frac{4c}{5a}$

54. $\frac{3a}{5}, \frac{6a^2}{10a}$

55. $\frac{2a}{3b}, \frac{10a^2b}{15ab^2}$

56. **Challenge** Jason believes that if the numerator or the denominator of a fraction is prime, then the fraction is in simplest form. Explain why this is not always true.

Mixed Review

In Exercises 57–59, evaluate the expression. Justify each step. (Lesson 2.6)

57. $-12 + 46 - 18$

58. $10 \cdot (-25) \cdot 0$

59. $\frac{1}{3} \cdot (20 \cdot 15)$

60. Find the greatest common factor of $3x^3y$ and $6x^2y^2$. (Lesson 4.2)

Basic Skills Use a protractor to draw an angle with the given measure.

61. 53°

62. 97°

63. 145°

Test-Taking Practice

64. **Multiple Choice** Mr. Wilkens has attended 18 of his son's 24 basketball games. What fraction of the games has he attended?

A. $\frac{3}{8}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

65. **Multiple Choice** Which pair of fractions are equivalent?

F. $\frac{6}{10}, \frac{9}{25}$

G. $\frac{3}{8}, \frac{15}{35}$

H. $\frac{14}{21}, \frac{24}{36}$

I. $\frac{2}{5}, \frac{5}{20}$

Make a List

Guess, Check, and Revise

Look for a Pattern

Draw a Diagram

Act It Out

Make a List

Work Backward

Solve a Simpler Problem

Problem Mia is trying to remember Jasmine's phone number. She knows the first three digits are 889, but she is confused about the last four numbers. Mia knows the last four digits are a 3, 4, 5, and 6, but she cannot recall their correct order. How many possibilities are there for Jasmine's phone number?

1 Read and Understand

Read the problem carefully.

You know the last four digits of a phone number, but not their order. You need to find how many ways these numbers can be arranged.

2 Make a Plan

Decide on a strategy to use.

One way to solve the problem is to make a list of all the possible four-digit numbers. Then you can use the list to count the number of possibilities for a phone number.

3 Solve the Problem

Reread the problem and make a list.

First, list all of the four-digit numbers that begin with the number 3.

3456 3465 3546 3564 3645 3654

Similarly, list all of the four-digit numbers that begin with the numbers 4, 5, and 6.

4356 4365 4536 4563 4635 4653

5346 5364 5436 5463 5634 5643

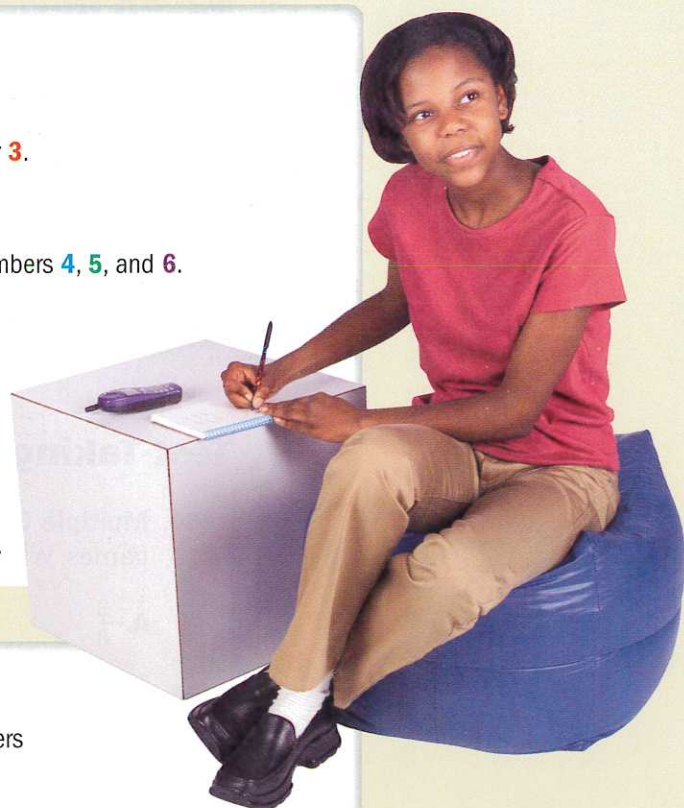
6345 6354 6435 6453 6534 6543

Now count all of the four-digit numbers in the list.

ANSWER There are 24 possibilities for Jasmine's phone number.

4 Look Back

Double-check your list to make sure you didn't repeat any numbers or forget a number.



Practice the Strategy

Use the strategy *make a list*.

- Baseball** The first team to win 3 out of 5 possible games will be the winner of a baseball tournament. In how many ways can a team win 3 out of 5 games?
- Telephone Prefixes** A certain town can use the digits 0, 1, 2, 6, 7, and 8 for its telephone prefixes. How many three-digit telephone prefixes are possible for this town if each digit can be used only once in a prefix and the first digit cannot be a 1 or a 0?
- Number Cube** Derek rolled 2 number cubes and added the two numbers on the top of each cube. List all the possible sums. In how many different ways can he roll a sum of 7?
- Tennis** Three boys and three girls sign up for a mixed doubles tennis tournament. In mixed doubles a team consists of one boy and one girl. How many mixed doubles teams can be made?
- Checkbook Covers** A bank has checkbook covers that are either pocket or desk size. The covers can be white, black, red, or tan. The customer's name will be stamped on the cover in gold or silver. How many different choices are available for checkbook covers?
- Summer Job** A student was hired by a city's maintenance department to paint parking space numbers in a city parking lot. Each digit had to be painted separately, and the student earned 20 cents per digit. The student painted parking space numbers from 1 to 225. How much did the student earn?



Mixed Problem Solving

Use any strategy to solve the problem.

- Car Show** The manager of a mall is asked to rope off a rectangular section of the parking lot for a car show. The area roped off is 250 feet by 300 feet. Posts are to be placed every 25 feet around the lot. How many posts are needed?
 - Consultant Fees** Two consultants were hired by a company. The total consultant fees were \$12,500. If one consultant had earned \$500 less, each consultant would have been paid the same. How much did each consultant earn?
 - Gardening** Joyce has planted a kudzu vine in her yard. The kudzu vine is a fast growing Japanese plant that was brought to the United States in 1876. Use the information in the table below to predict how long Joyce's vine will be after 2 weeks.
- | | | | | |
|--------------------------------------|----|----|----|----|
| Days | 1 | 2 | 3 | 4 |
| Length of kudzu vine (inches) | 12 | 24 | 36 | 48 |
- Video Games** Chris buys a video game and two T-shirts for \$44 at the mall. One week later he buys two more video games and a T-shirt for \$52. Each video game has the same price, and each T-shirt has the same price. How much does one video game cost?

LESSON 4.4

Least Common Multiple

BEFORE

You found the greatest common factor of two numbers.

Now

You'll find the least common multiple of two numbers.

WHY?

So you can plan your weekly schedule, as in Ex. 26.

Word Watch

multiple, p. 186
common multiple, p. 186
least common multiple (LCM), p. 186

In the Real World

Animal Clinic A veterinarian at an animal clinic is on call every four days. Today is Saturday, and the vet is on call. In how many more days will the vet be on call on a Saturday again? You will see how to solve this problem in Example 1.



A **multiple** of a number is the product of the number and any nonzero whole number. A multiple that is shared by two or more numbers is a **common multiple**. The least of the common multiples of two or more whole numbers is the **least common multiple (LCM)**.

EXAMPLE 1 Finding the Least Common Multiple

The veterinarian described above is on call every 4 days. A Saturday occurs every 7 days. To determine the next Saturday the vet will be on call, find the least common multiple of 4 and 7.

Method 1: Make a list.

List the multiples of each number.

Multiples of 4: 4, 8, 12, 16, 20, 24, **28**, 32, 36, 40, 44, ...

Multiples of 7: 7, 14, 21, **28**, 35, 42, 49, 56, ...

The LCM of 4 and 7 is 28.

Method 2: Use prime factorization.

Write the prime factorization of each number.

$$4 = 2^2 \quad 7 = 7$$

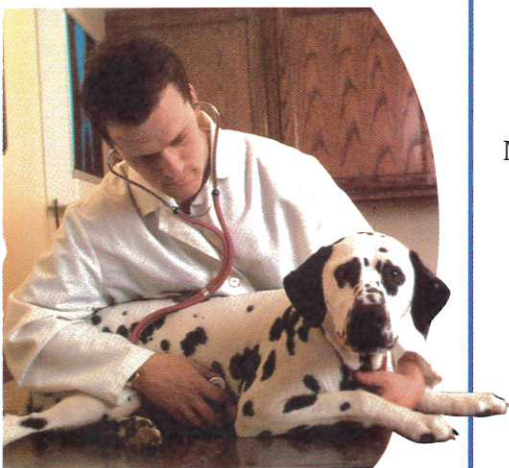
Write the product of the highest power of each prime number in the prime factorizations.

$$2^2 \cdot 7 = 28$$

The LCM of 4 and 7 is 28.



ANSWER In 28 days, the veterinarian will be on call on a Saturday.



EXAMPLE 2 Finding the Least Common Multiple

Find the LCM of 32, 96, and 120 using prime factorization.

Solution

Write the prime factorization of each number.

$$32 = 2^5$$

$$96 = 2^5 \cdot 3$$

$$120 = 2^3 \cdot 3 \cdot 5$$

Write the product of the highest power of each prime number in the prime factorizations.

$$2^5 \cdot 3 \cdot 5 = 480$$

ANSWER The LCM of 32, 96, and 120 is 480.

Your turn now Find the least common multiple of the numbers.

1. 6, 15

2. 4, 20

3. 12, 28

4. 24, 36, and 72

Method 2 of Example 1 is also useful for finding the least common multiples of monomials.

EXAMPLE 3 Finding the LCM of Monomials

Find the LCM of $6x^2y$ and $9x^4z$.

Solution

Factor each expression using exponents.

$$6x^2y = 2 \cdot 3 \cdot x^2 \cdot y$$

$$9x^4z = 3^2 \cdot x^4 \cdot z$$

Find the product of the highest power of each factor, including the variables.

$$2 \cdot 3^2 \cdot x^4 \cdot y \cdot z = 18x^4yz$$

ANSWER The LCM of $6x^2y$ and $9x^4z$ is $18x^4yz$.

Your turn now Find the least common multiple of the monomials.

5. $8x^3$, $20x^7$

6. $12y^4$, $36y^8$

7. $4ab^2$, $10a^2b$

8. $6m^3np^2$, $8mp^3$

Getting Ready to Practice

Vocabulary Copy and complete the statement.

1. A(n) ? of 6 and 9 is 54. 2. The ? of 6 and 9 is 18.

Matching Match the pair of numbers with its LCM.

3. 36, 18 4. 45, 75 5. 6, 18 6. 42, 105
 A. 18 B. 36 C. 210 D. 225

7. **Find the Error** Describe and correct the error in the solution.

Find the LCM of 12 and 24.

$$12 = 2 \cdot 2 \cdot 3 \quad 24 = 2 \cdot 2 \cdot 2 \cdot 3$$

The LCM is $2 \cdot 2$, or 4.



HELP with Homework

Example Exercises

- 1 8–11, 25, 26
 2 12–19
 3 20–23

Online Resources
 CLASSZONE.COM

- More Examples
- eTutorial Plus

Practice and Problem Solving

List the first few multiples of each number. Then use the lists to find the LCM of the numbers.

8. 4, 6 9. 6, 21 10. 8, 10 11. 10, 15

Write the prime factorization of the numbers. Then find their LCM.

12. 36, 90 13. 17, 57 14. 90, 108 15. 125, 500
 16. 6, 8, 12 17. 8, 16, 32 18. 6, 15, 45 19. 20, 24, 60

Find the LCM of the monomials.

20. $5ab, 7ab^2$ 21. $7s^3t, 49st^2$
 22. $4x^3y^3, 18xy^5$ 23. $24c^2d^3, 60c^2d^6$

24. **Writing** Could you find the *greatest* common multiple of two numbers? Explain your reasoning.

25. **Traffic Lights** One traffic light turns red every 45 seconds. Another traffic light turns red every 60 seconds. Both traffic lights just turned red. In how many seconds will they turn red at the same time again?

26. **Schedule** Your class schedule changes on a three-day rotation. Every three days you have math class during the last class period of the day. This week, you have math class the last period on Friday. In how many more school days will you have math class the last period on Friday?





Find the LCM of the numbers using prime factorization.

27. 160, 432 28. 144, 576 29. 21, 36, 57 30. 18, 54, 84
31. 30, 75, 100 32. 36, 54, 72 33. 10, 12, 30, 60 34. 21, 42, 63, 105

Find the LCM of the monomials.

35. $24x^4y$, $30y^7$ 36. $17m^3n^3$, $9m^2n^6$ 37. $45gh^5k^3$, $33g^4hk^3$

38. **Lasagna** Zoe is making lasagna for a family reunion. Her recipe calls for twelve noodles for each batch of lasagna. One box of lasagna noodles contains 14 noodles. What is the least number of batches of lasagna that Zoe can make without having any noodles left over?
39. **Swimming** Will swims one lap in 160 seconds, while Martin swims one lap in 180 seconds. The boys start their laps at the same time from the same side of the pool and maintain their pace. When will they both be at their starting place at the same time again? Write your answer in minutes and seconds.
40. **Writing** You are asked to find the LCM of two numbers. One of the numbers is a factor of the other number. Is there a shortcut to finding their LCM? Explain.
41. **Challenge** Could the GCF of two different numbers also be the LCM of those numbers? Explain.

Mixed Review

42. Rebecca ran on her treadmill at 5.6 miles per hour for one half hour. How many miles did Rebecca run? (*Lesson 1.6*)
43. Simplify the expression $7x + 9 + 12x + 11 + 2y$ by combining like terms. (*Lesson 2.7*)
44. Find the greatest common factor of 121 and 187. (*Lesson 4.2*)

Basic Skills Find the sum.

45. $24.63 + 49.07$ 46. $14.125 + 16.8$ 47. $33.87 + 100.9$

Test-Taking Practice

48. **Multiple Choice** What is the prime factorization of 72?
A. $2^2 \cdot 3 \cdot 6$ B. $2 \cdot 6^2$ C. $2^3 \cdot 3^2$ D. $2^2 \cdot 3^2 \cdot 6$
49. **Short Response** A teacher can arrange a class into groups of 2, 5, or 6 students with no one left out. What is the least number of students that the teacher can have in class to do this? Explain how you found your answer.

Notebook Review



Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

prime number, p. 169
composite number, p. 169
prime factorization, p. 169
factor tree, p. 169

monomial, p. 170
common factor, p. 173
greatest common factor (GCF), p. 173
relatively prime, p. 174
simplest form, p. 179

equivalent fractions, p. 179
multiple, p. 186
common multiple, p. 186
least common multiple (LCM), p. 186

Use Your Vocabulary

1. Copy and complete: A factor tree can be used to find the ? of a number.

4.1 Can you write a prime factorization?



EXAMPLE Write the prime factorization of 504.

$$504 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7, \text{ or } 2^3 \cdot 3^2 \cdot 7$$

ANSWER The prime factorization of 504 is $2^3 \cdot 3^2 \cdot 7$.



Write the prime factorization of the number.

2. 40

3. 7

4. 85

5. 120

4.2 Can you find the GCF of two numbers?



EXAMPLE Find the GCF of 36 and 60.

$$36 = 2^2 \cdot 3^2$$

$$60 = 2^2 \cdot 3 \cdot 5$$

The common factors are 2^2 and 3. So, the GCF is $2^2 \cdot 3$, or 12.

ANSWER The GCF of 36 and 60 is 12.



Find the GCF of the numbers or monomials.

6. 48, 80

7. 60, 100

8. $14a^3$, $21a$

9. $20y^4$, $60y^5$

4.3 Can you write a fraction in simplest form?



EXAMPLE Write $\frac{48}{72}$ in simplest form.

$$\frac{48}{72} = \frac{48 \div 24}{72 \div 24} = \frac{2}{3}$$



Write the fraction in simplest form.

10. $\frac{15}{45}$

11. $\frac{12}{80}$

12. $\frac{9ab}{27a}$

13. $\frac{18n^3}{54n}$

4.4 Can you find the LCM of two numbers?



EXAMPLE Find the LCM of 20 and 48.

$$20 = 2^2 \cdot 5$$

$$48 = 2^4 \cdot 3$$

ANSWER The LCM of 20 and 48 is $2^4 \cdot 3 \cdot 5$, or 240.



Find the LCM of the numbers or monomials.

14. 28, 42

15. 54, 90

16. $10cd$, $25c^2$

17. $9n^3$, $12n^2$

Stop and Think

about Lessons 4.1–4.4



18. **Writing** Explain the difference between listing the factors of a number and finding the prime factorization of a number.

Review Quiz 1

Find the GCF of the numbers or monomials.

1. 24, 90

2. 36, 72, 108

3. $20c^3$, $48c^2$

4. $64m^2$, $80m^5$

Find the LCM of the numbers or monomials.

5. 88, 99

6. 36, 96

7. $7xy$, $21y^3$

8. $6ab^2$, $30ab$

Tell whether the fractions are equivalent.

9. $\frac{9}{27}$, $\frac{60}{180}$

10. $\frac{39}{91}$, $\frac{42}{56}$

11. $\frac{40}{48}$, $\frac{70}{84}$

12. $\frac{108}{120}$, $\frac{189}{210}$

13. **Supermarket** A supermarket gives every tenth customer a coupon and every twenty-fifth customer a gift. Which of the first 200 customers receive both a coupon and a gift?

LESSON 4.5

Comparing Fractions and Mixed Numbers

BEFORE

You compared and ordered integers.

Now

You'll compare and order fractions and mixed numbers.

WHY?

So you can determine the greater fraction of games won, as in Ex. 28.

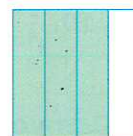
Word Watch

least common denominator (LCD), p. 192

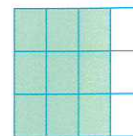
You can use models to compare the fractions $\frac{2}{3}$ and $\frac{3}{4}$.



=



=



$$\frac{2}{3} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$$

In the diagram above, $\frac{8}{12} < \frac{9}{12}$, so $\frac{2}{3} < \frac{3}{4}$.

The **least common denominator (LCD)** of two or more fractions is the least common multiple of the denominators. You can compare fractions by using the least common denominator to write equivalent fractions.

EXAMPLE 1 Comparing Fractions Using the LCD

Compare $\frac{3}{8}$ and $\frac{5}{12}$.

- (1) Find the least common denominator of the fractions.
The LCM of 8 and 12 is 24, so the least common denominator is 24.

- (2) Use the least common denominator to write equivalent fractions.

$$\frac{3}{8} = \frac{3 \cdot 3}{8 \cdot 3} = \frac{9}{24}$$

$$\frac{5}{12} = \frac{5 \cdot 2}{12 \cdot 2} = \frac{10}{24}$$

- (3) Compare the numerators: $9 < 10$, so $\frac{9}{24} < \frac{10}{24}$.

ANSWER Because $\frac{9}{24} < \frac{10}{24}$, you can write $\frac{3}{8} < \frac{5}{12}$.

HELP with Solving

You can write equivalent fractions by multiplying or dividing the numerator and denominator by the same nonzero number.

Your turn now Copy and complete the statement with $<$, $>$, or $=$.

1. $\frac{2}{3} ? \frac{5}{8}$

2. $\frac{2}{4} ? \frac{15}{20}$

3. $\frac{3}{10} ? \frac{2}{4}$

4. $\frac{9}{16} ? \frac{11}{18}$

To compare or order improper fractions and mixed numbers, first write any mixed numbers as improper fractions.

HELP with Review

For help with writing mixed numbers as improper fractions, see p.707.

EXAMPLE 2 Ordering Fractions and Mixed Numbers

Order the numbers $4\frac{7}{16}$, $\frac{19}{4}$, and $\frac{35}{8}$ from least to greatest.

- (1) Find the least common denominator of the fractions.

The LCM of 16, 4, and 8 is 16, so the LCD is 16.

- (2) Use the least common denominator to write equivalent fractions.

$$4\frac{7}{16} = \frac{4 \cdot 16 + 7}{16} = \frac{71}{16} \quad \frac{19}{4} = \frac{19 \cdot 4}{4 \cdot 4} = \frac{76}{16} \quad \frac{35}{8} = \frac{35 \cdot 2}{8 \cdot 2} = \frac{70}{16}$$

- (3) Compare the numerators: $70 < 71$, and $71 < 76$, so $\frac{70}{16} < \frac{71}{16}$ and $\frac{71}{16} < \frac{76}{16}$.

ANSWER From least to greatest, the numbers are $\frac{35}{8}$, $4\frac{7}{16}$, and $\frac{19}{4}$.

EXAMPLE 3 Comparing Mixed Numbers

Orangutans A female orangutan is about $3\frac{1}{2}$ feet tall. A male orangutan is about $3\frac{2}{5}$ feet tall. Which of the two orangutans is taller?

Solution

The LCM of 2 and 5 is 10, so the least common denominator is 10.

Use the least common denominator to write equivalent fractions.

$$3\frac{1}{2} = \frac{3 \cdot 2 + 1}{2} = \frac{7}{2} \quad 3\frac{2}{5} = \frac{3 \cdot 5 + 2}{5} = \frac{17}{5}$$

$$\frac{7}{2} = \frac{7 \cdot 5}{2 \cdot 5} = \frac{35}{10} \quad \frac{17}{5} = \frac{17 \cdot 2}{5 \cdot 2} = \frac{34}{10}$$

Because $35 > 34$, you can write $\frac{35}{10} > \frac{34}{10}$.

ANSWER The female orangutan is taller.

What do you think?

Zoology



Orangutans

An orangutan's arms are about two thirds as long as its height. How long would the arms of a 3 foot orangutan be?

Your turn now Copy and complete the statement with $<$, $>$, or $=$.

5. $\frac{16}{5} ? 3\frac{1}{3}$

6. $1\frac{4}{5} ? \frac{21}{12}$

7. $-2\frac{2}{3} ? -4\frac{5}{6}$

8. Order the numbers $2\frac{7}{9}$, $2\frac{5}{12}$, and $\frac{11}{4}$ from least to greatest.



Getting Ready to Practice

1. **Vocabulary** Copy and complete: The least common denominator of two fractions is the ? of their denominators.

Find the least common denominator of the fractions.

2. $\frac{1}{2}, \frac{2}{3}$

3. $\frac{3}{4}, \frac{7}{20}$

4. $\frac{11}{24}, \frac{5}{6}$

5. $\frac{5}{12}, \frac{7}{18}$

Copy and complete the statement with $<$, $>$, or $=$.

6. $\frac{3}{5} ? \frac{7}{10}$

7. $\frac{7}{18} ? \frac{5}{9}$

8. $\frac{24}{32} ? \frac{3}{4}$

9. $\frac{5}{8} ? \frac{7}{12}$

10. **Guided Problem Solving** Sarah walks two thirds of a mile to school every day. Amy walks five eighths of a mile to school. Whose walk to school is longer?

- (1) Find the least common denominator.
- (2) Rewrite both fractions using the LCD.
- (3) Use your answer to find whose walk is longer.

Practice and Problem Solving

Copy and complete the statement with $<$, $>$, or $=$.

11. $3\frac{1}{4} ? \frac{13}{12}$

12. $\frac{31}{6} ? 5\frac{1}{6}$

13. $\frac{5}{11} ? \frac{42}{55}$

14. $2\frac{4}{5} ? \frac{7}{3}$

15. $\frac{165}{36} ? 4\frac{5}{12}$

16. $\frac{11}{18} ? \frac{9}{14}$

Order the numbers from least to greatest.

17. $\frac{1}{2}, \frac{1}{8}, \frac{3}{4}, \frac{5}{16}$

18. $1\frac{1}{2}, \frac{5}{4}, \frac{11}{6}$

19. $\frac{5}{3}, \frac{35}{15}, 2\frac{2}{5}, \frac{15}{16}$

20. **Commercials** During a 30 minute TV show, there are 8 minutes of commercials. During a 2 hour movie, there are 31 minutes of commercials. Write each commercial time as a fraction of the total time. Which TV program has a greater fraction of commercial time?
21. **Writing** Explain how comparing fractions with like denominators differs from comparing fractions with unlike denominators.

HELP
with Homework
Example Exercises

- | | |
|---|---------------|
| 1 | 11-16 |
| 2 | 17-19, 22, 23 |
| 3 | 11-16, 20 |


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HELP with Solving

A negative fraction can be written in various ways. For example,

$$-\frac{a}{b} = \frac{a}{-b} = \frac{-a}{b}$$



Write the numbers in order from least to greatest.

22. $7\frac{1}{4}, \frac{31}{4}, \frac{63}{8}, \frac{47}{6}, 7\frac{19}{24}$

23. $-\frac{34}{3}, -11\frac{7}{12}, -11\frac{17}{48}, -\frac{23}{2}, -\frac{47}{4}$

Copy and complete the statement with $<$, $>$, or $=$ by first comparing each fraction to $\frac{1}{2}$.

24. $\frac{25}{50} ? \frac{37}{74}$

25. $\frac{17}{30} ? \frac{10}{33}$

26. $\frac{23}{100} ? \frac{19}{36}$

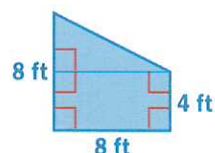
27. **Critical Thinking** In Exercises 24–26, does it help to compare each number to $\frac{1}{2}$ first? Will this step always work? If not, could you use another fraction to help make comparisons?

28. **Little League** Teams from California have played in the Little League World Series 19 times and won the championship 5 times. Texas teams have appeared in 7 Little League World Series and won twice. Which state has won a greater fraction of their World Series games?

29. **Challenge** Consider the fractions $\frac{1}{2x}$ and $\frac{1}{x}$. What is their LCD? Write each fraction using the LCD.

Mixed Review

30. Find the area of the figure at the right. (Lesson 3.5)



31. Write the prime factorization of 336. (Lesson 4.1)

32. **Basic Skills** Find the quotient of 1998 and 42. Round your answer to the nearest thousandth.

Test-Taking Practice

33. **Multiple Choice** Which list of fractions is written correctly in order from least to greatest?

A. $\frac{12}{18}, \frac{13}{30}, \frac{8}{15}$

B. $\frac{4}{18}, \frac{9}{15}, \frac{18}{27}$

C. $\frac{6}{10}, \frac{4}{18}, \frac{16}{24}$

D. $\frac{7}{11}, \frac{7}{8}, \frac{15}{25}$

34. **Multiple Choice** In a class of 32 people, 28 were at school, so $\frac{28}{32}$ of the class was present. What is another way to express this number?

F. $\frac{4}{8}$

G. $\frac{24}{28}$

H. $\frac{7}{8}$

I. $\frac{15}{16}$

LESSON 4.6

Rules of Exponents

BEFORE

You multiplied and divided numerical expressions.

Now

You'll multiply and divide expressions with exponents.

WHY?

So you can compare the memory in two computers, as in Exs. 49–50.

Word Watch

Review Words

exponent, p. 20

power, p. 20

Activity

Using patterns to discover rules for multiplying powers.

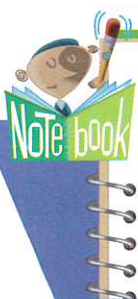
- 1 Copy and complete the table.

Expression	Expanded Expression	Number of Factors	Product as a Power
$2^2 \cdot 2^4$	$(2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$	6	2^6
$3^3 \cdot 3^1$	$(3 \cdot 3 \cdot 3) \cdot 3$?	$3^?$
$7^2 \cdot 7^3$?	?	?

- 2 How are the exponents in the first and last columns related?
 3 Write the product $6^5 \cdot 6^{11}$ as a single power.

As you saw in the activity, you can expand expressions to find their product. The following equation suggests a rule for multiplying powers with the same base when the exponents are integers.

$$a^4 \cdot a^2 = \overbrace{(a \cdot a \cdot a \cdot a)}^{4 \text{ factors}} \cdot \overbrace{(a \cdot a)}^{2 \text{ factors}} = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdot a}_{6 \text{ factors}} = a^{4+2} = a^6$$



Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Algebra $a^m \cdot a^n = a^{m+n}$ **Numbers** $5^6 \cdot 5^3 = 5^{6+3} = 5^9$

EXAMPLE 1

Using the Product of Powers Property

$$\begin{aligned} x^4 \cdot x^7 &= x^{4+7} \\ &= x^{11} \end{aligned}$$

Product of powers property

Add exponents.

Watch Out!



Remember that numbers raised to the first power are usually written without an exponent. For example, $3 = 3^1$.

EXAMPLE 2 Using the Product of Powers Property

$$\begin{aligned}3^2x^2 \cdot 3x^3 &= (3^2 \cdot 3) \cdot (x^2 \cdot x^3) \\&= 3^{2+1} \cdot x^{2+3} \\&= 3^3x^5 \\&= 27x^5\end{aligned}$$

Commutative property of multiplication

Product of powers property

Add exponents.

Evaluate the power.

The following equation suggests a rule for dividing powers with the same base when the exponents are integers.

$$\frac{a^5}{a^3} = \frac{\overbrace{a \cdot a \cdot a \cdot a \cdot a}^{5 \text{ factors}}}{\underbrace{a \cdot a \cdot a}_{3 \text{ factors}}} = \frac{a \cdot a \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}}} = \overbrace{a \cdot a}^{2 \text{ factors}} = a^{5-3} = a^2$$



Quotient of Powers Property

Words

To divide two powers with the same nonzero base, subtract the exponent of the denominator from the exponent of the numerator.

Algebra

$$\frac{a^m}{a^n} = a^{m-n}$$

Numbers

$$\frac{4^7}{4^4} = 4^{7-4} = 4^3$$

EXAMPLE 3 Using the Quotient of Powers Property

Simplify the expression. Write your answer as a power.

a. $\frac{x^{12}}{x^7} = x^{12-7}$

Quotient of powers property

$$= x^5$$

Subtract exponents.

b. $\frac{9^7}{9^3} = 9^{7-3}$

Quotient of powers property

$$= 9^4$$

Subtract exponents.

Your turn now

Simplify the expression. Write your answer as a power.

1. $a^6 \cdot a^4$

2. $2^3 \cdot 2^4$

3. $\frac{a^6}{a^4}$

4. $\frac{10^9}{10^6}$

Watch Out!



The bases of the powers must be the same to use the product or quotient property. In part (b) of Example 4, you cannot simplify the numerator any further because the bases, x and y , are different.

EXAMPLE 4 Simplifying Fractions with Powers

$$\begin{aligned}\text{a. } \frac{y^4 \cdot y}{y^3} &= \frac{y^5}{y^3} \\ &= y^{5-3} \\ &= y^2\end{aligned}$$

Simplify numerator using product of powers property.

Quotient of powers property

Subtract exponents.

$$\begin{aligned}\text{b. } \frac{xy^4}{y^3} &= xy^{4-3} \\ &= xy\end{aligned}$$

Quotient of powers property

Subtract exponents.

Your turn now

Simplify the expression. Write your answer as a power.

5. $\frac{q^3 \cdot q^5}{q^4}$

6. $\frac{4^3 \cdot 4^{12}}{4^5}$

7. $\frac{a^2 b^8}{b^2}$

8. $\frac{x^5 y^{11}}{y^5}$

4.6

Exercises

More Practice, p. 730



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Getting Ready to Practice

Vocabulary Copy and complete the statement.

- Three is the ? of the expression 3^4 .
- Seven is the ? of the expression 4^7 .

Tell whether the product of powers property can be used to simplify the expression.

3. $9^3 \cdot 9^4$

4. $7^2 \cdot 2^7$

5. $r^6 \cdot s^6$

6. $n^5 \cdot n^8$

Simplify the expression. Write your answer as a power.

7. $4^2 \cdot 4^4$

8. $8 \cdot 8^3$

9. $a^5 \cdot a^7$

10. $b^9 \cdot b^9$

11. $\frac{c^6}{c^5}$

12. $\frac{5^8}{5^4}$

13. $\frac{8^7}{8^2}$

14. $\frac{d^8}{d}$

15. **Find the Error** Describe and correct the error in the solution.



$$\begin{aligned}2^2 \cdot 2^4 &= (2 \cdot 2)^{2+4} \\ &= 4^6\end{aligned}$$

HELP with Homework

Example Exercises

- | | |
|---|-------------------------|
| 1 | 16–19, 24–27,
32, 33 |
| 2 | 36–39 |
| 3 | 20–23, 28–31,
34, 35 |
| 4 | 40–43 |



- More Examples
- eTutorial Plus

Practice and Problem Solving

Simplify the expression.

- | | | | |
|---------------------|--------------------------|--------------------------|-----------------------------|
| 16. $u^7 \cdot u^8$ | 17. $v^2 \cdot v^{10}$ | 18. $b^9 \cdot b^6$ | 19. $m^{11} \cdot m^8$ |
| 20. $\frac{a^4}{a}$ | 21. $\frac{x^{10}}{x^6}$ | 22. $\frac{w^{15}}{w^9}$ | 23. $\frac{y^{20}}{y^{18}}$ |

Simplify the expression. Write your answer as a power.

- | | | | |
|-----------------------------|---------------------------|--------------------------|-----------------------|
| 24. $3^2 \cdot 3^4$ | 25. $(-4)^2 \cdot (-4)^3$ | 26. $5^4 \cdot 5$ | 27. $7^2 \cdot 7^2$ |
| 28. $\frac{(-7)^7}{(-7)^4}$ | 29. $\frac{2^{13}}{2^3}$ | 30. $\frac{6^{11}}{6^8}$ | 31. $\frac{9^8}{9^4}$ |

Determine the number that correctly completes the equation.

- | | | | |
|------------------------------|---------------------------|-----------------------------|--------------------------------|
| 32. $2^3 \cdot 2^? = 2^{11}$ | 33. $5^4 \cdot 5^? = 5^9$ | 34. $\frac{8^7}{8^?} = 8^3$ | 35. $\frac{12^?}{12^5} = 12^4$ |
|------------------------------|---------------------------|-----------------------------|--------------------------------|

Simplify the expression.

- | | | | |
|---------------------------|---------------------------------|-----------------------------|-----------------------------------|
| 36. $3a^3 \cdot 3a^2$ | 37. $2y^3 \cdot 2y^2$ | 38. $3^2x^5 \cdot 3^3x^4$ | 39. $4a^3b^4 \cdot 4^2a^4b^6$ |
| 40. $\frac{p^5q^9}{pq^5}$ | 41. $\frac{z^6 \cdot z^3}{z^4}$ | 42. $\frac{3^3m^9}{3^2m^5}$ | 43. $\frac{5^5n^{15}}{5^3n^{12}}$ |

44. **Critical Thinking** Write a quotient that simplifies to x^4y^4 .

Measurement In Exercises 45–48, use the table. It shows the number of meters in some metric measures written as powers of ten.

Metric Units	
Unit	Meters
Yottameter	10^{24}
Zettameter	10^{21}
Exameter	10^{18}
Petameter	10^{15}
Terameter	10^{12}
Gigameter	10^9
Megameter	10^6
Kilometer	10^3
Decameter	10^1

45. How many kilometers are in a petameter?
46. How many gigameters are in a zettameter?
47. How many terameters are in a yottameter?
48. How many megameters are in an exameter?

Computers In the 1970s and early 1980s, computer random access memory was measured in kilobytes (KB) and could be added only in quantities equal to a power of 2. In Exercises 49 and 50, how many times more memory did the newer computer have?

- | | |
|------------------------------------|------------------------------------|
| 49. 1979: 2^3 KB; 1980: 2^5 KB | 50. 1982: 2^6 KB; 1987: 2^9 KB |
|------------------------------------|------------------------------------|



Astronomy



Measurement

The distance to the Andromeda Galaxy is 21 quintillion kilometers, which is 21 followed by 18 zeros. How many exameters are in 21 quintillion kilometers?

Challenge Evaluate the expression.

51. $(3^2 \cdot 3)^2$ 52. $(2^0 \cdot 2^2)^3$ 53. $\left(\frac{4^7}{4^5}\right)^2$ 54. $\left(\frac{5^8}{5^7}\right)^4$

Mixed Review

Evaluate the expression. (Lesson 1.4)

55. $(4 \times 3)^2 + 13$ 56. $405 \div (14 - 11)^4$ 57. $96 \div 2^5 \times 6$

Copy and complete the statement with $<$, $>$, or $=$. (Lesson 4.5)

58. $\frac{5}{2} ? \frac{15}{6}$ 59. $\frac{9}{24} ? \frac{5}{16}$ 60. $\frac{5}{8} ? \frac{7}{11}$

Test-Taking Practice

61. **Multiple Choice** What is the value of $3^2 \cdot 3^2 - 4^3$?
A. -46 B. -17 C. 17 D. 46
62. **Multiple Choice** What is another expression for $\frac{a^9 \cdot a^4}{a^5}$?
F. a^4 G. a^5 H. a^8 I. a^{13}



BRAIN GAME

Mix and Match

Materials: cards marked from 1 to 6

Number of Players: 2 or 3 players

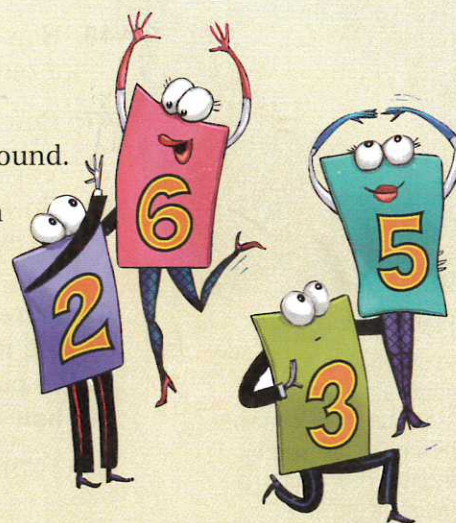
Winning Strategy: Make the largest number possible in each round.

Mix the cards and place them face down on a flat surface. Each player takes two cards. Then using only the two cards, make the largest number possible. For example, if you picked the cards 2 and 6, you could make the following numbers:

26 62 2^6 6^2

The largest number you can make is 2^6 , or 64. So, you would score 64 points for that round. After each round, reshuffle the cards to play another round.

The player with the highest score after three rounds wins.



LESSON 4.7

Negative and Zero Exponents

BEFORE

You simplified expressions with positive exponents.

Now

You'll simplify expressions with negative exponents.

WHY?

So you can describe very small objects, as in Ex. 31.

In the Real World

Word Watch

Review Words

exponent, p. 20
common factor, p. 173

Strobes The picture at the right was taken using a strobe light. The flash of the strobe light lasted about 1 microsecond. How can you write this time in seconds as a power of ten? You will see how to solve this problem in Example 1.



You have seen two methods for evaluating expressions involving division of powers.

Divide out common factors.

$$\frac{x^5}{x^7} = \frac{\cancel{x}^1 \cdot \cancel{x}^1 \cdot \cancel{x}^1 \cdot \cancel{x}^1 \cdot \cancel{x}^1}{\cancel{x}_1 \cdot \cancel{x}_1 \cdot \cancel{x}_1 \cdot \cancel{x}_1 \cdot \cancel{x}_1 \cdot x \cdot x} = \frac{1}{x^2}$$

Quotient of powers property

$$\frac{x^5}{x^7} = x^{5-7} = x^{-2}$$

So $\frac{1}{x^2} = x^{-2}$, which suggests the definition for negative exponents.



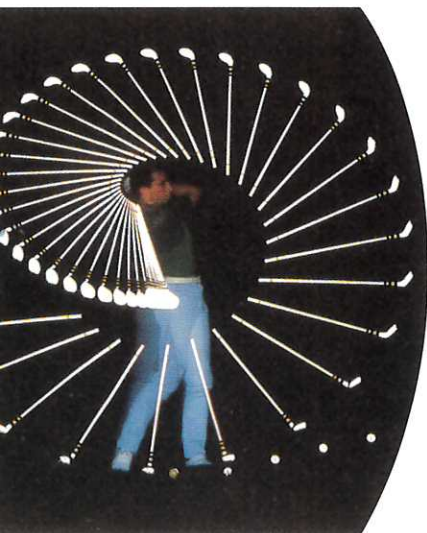
Negative Exponents

Words For any integer n and any number $a \neq 0$,

$$a^{-n} \text{ is equal to } \frac{1}{a^n}.$$

Algebra $a^{-n} = \frac{1}{a^n}$

Numbers $2^{-3} = \frac{1}{2^3}$



EXAMPLE 1 Using a Negative Exponent

The flash above lasts 1 microsecond, or $\frac{1}{1,000,000}$ second.

$$\frac{1}{1,000,000} = \frac{1}{10^6}$$

Write 1,000,000 as 10^6 .

$$= 10^{-6}$$

Definition of negative exponent

ANSWER One flash of a strobe light lasts about 10^{-6} second.

EXAMPLE 2 Evaluating a Numerical Expression

$$5^2 \cdot 5^{-5} = 5^{2+(-5)}$$

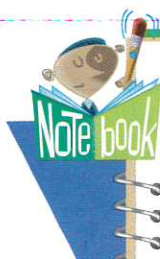
Product of powers property

$$= 5^{-3}$$

Simplify.

$$= \frac{1}{5^3} = \frac{1}{125}$$

Use definition of negative exponent and evaluate power.

**Zero Exponents****Algebra** If a is a nonzero number, then $a^0 = 1$.**Numbers** $2^0 = 1$ **Watch Out!**

In an expression such as $-2n^0$ and $4n^{-5}$, the exponent is applied only to the variable, not to the coefficient.

EXAMPLE 3 Simplifying Variable Expressions**Simplify.** Write the expression using only positive exponents.

a. $-2n^0 = -2 \cdot n^0$

Zero exponent applies only to n .

$$= -2 \cdot 1$$

Definition of zero exponent

$$= -2$$

Multiply.

b. $4n^{-5} = 4 \cdot n^{-5}$

Exponent applies only to n .

$$= 4 \cdot \frac{1}{n^5}$$

Definition of negative exponent

$$= \frac{4}{n^5}$$

Multiply.

c. $\frac{8x^{-3}}{x} = \frac{8 \cdot x^{-3}}{x^1}$

Exponent applies only to x .

$$= 8 \cdot x^{-3-1}$$

Quotient of powers property

$$= 8 \cdot x^{-4}$$

Simplify.

$$= \frac{8}{x^4}$$

Definition of negative exponent

Your turn now Evaluate the expression.

1. 7^{-2}

2. $(-2)^{-5}$

3. $6 \cdot 6^{-3}$

4. $10^{-5} \cdot 10^7$

Simplify. Write the expression using only positive exponents.

5. $-6m^{-1}$

6. $b^2 \cdot b^{-2}$

7. $\frac{5x^4}{x^7}$

8. $\frac{10a^{-3}}{a^4}$

Getting Ready to Practice

Vocabulary Determine whether the statement is *true* or *false*.

- The base of the expression 2^{-5} is 2.
- The exponent of the expression 2^{-5} is 5.

Evaluate the expression.

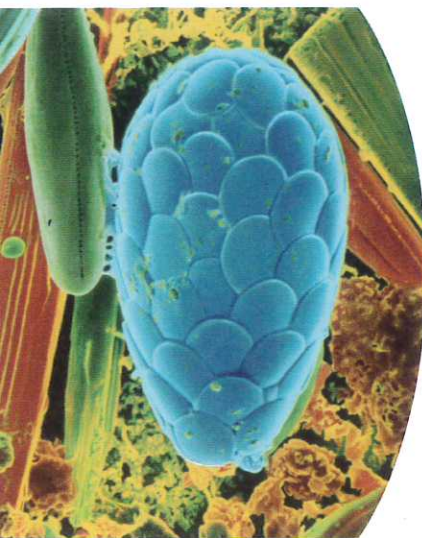
- 3^{-4}
- $(-4)^{-3}$
- $2^{-10} \cdot 2^6$
- 12^0

- Find the Error** Describe and correct the error in the solution.

$$5^{-3} = (-5)(-5)(-5) \\ = -125$$



- Biology** Plankton is made up of tiny plants (called phytoplankton) and tiny animals (called zooplankton). One type of phytoplankton may be as small as 0.2 micrometer. A micrometer is 10^{-6} meter. What part of a meter is this phytoplankton? Use a positive exponent to write your answer.



Single-celled alga

HELP

with Homework

Example Exercises

- | | |
|---|--------|
| 1 | 21, 31 |
| 2 | 9-12 |
| 3 | 13-20 |



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Practice and Problem Solving

Evaluate the expression.

- $(-6)^{-2}$
- $2 \cdot 2^{-6}$
- $5^4 \cdot 5^{-8}$
- 9^0

Simplify. Write the expression using only positive exponents.

- $m^{-9} \cdot m^5$
- $x^5 \cdot x^{-5}$
- $9n^{-3}$
- $c^{-1} \cdot c^{-2} \cdot c^{-4}$
- $b^3 \cdot b^{-4} \cdot b^{-5}$
- $\frac{4z^{-2}}{z^4}$
- $\frac{a^{-5}}{a^8}$
- $\frac{18r^{-6}}{3r^3}$

- Physics** Pressure is measured in units called *pascals*. This unit can be expressed as $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$. Write the unit without negative exponents.

Find the missing exponent.

- $(4x^5)^? = 1$
- $15a^? = \frac{15}{a^8}$
- $y^? \cdot y^4 = \frac{1}{y}$
- $\frac{x^{-3}}{x^?} = \frac{1}{x^{13}}$



- Writing** Your friend missed today's class. Write a note to show your friend how to simplify the expression $\frac{6a^{-3}}{a^3}$.



Measurement In Exercises 27–30, use the table. It shows the number of meters in some metric measures written as powers of ten.

27. How many picometers are in a decimeter?
28. How many yoctometers are in a micrometer?
29. How many nanometers are in a decimeter?
30. How many attometers are in a centimeter?
31. **Teddy Bear** In 1997, German teddy bear specialist Hanne Schramm made the smallest teddy bear in the world. It measures 0.47 inch or about 10 millimeters. How many nanometers are in 10 millimeters?

Metric Units	
Unit	Meter
Decimeter	10^{-1}
Centimeter	10^{-2}
Millimeter	10^{-3}
Micrometer	10^{-6}
Nanometer	10^{-9}
Picometer	10^{-12}
Attometer	10^{-18}
Yoctometer	10^{-24}

Critical Thinking In Exercises 32–33, copy and complete the statement using *always*, *sometimes*, or *never*.

32. A power with a negative exponent can ? be written as a fraction.
33. A power with a positive base and a negative exponent is ? negative.
34. **Challenge** Use the product of powers property to explain why $a^0 = 1$, where a is a nonzero number, makes sense.

Mixed Review

Simplify the expression. (Lessons 2.2–2.5)

35. $-18 + (-7)$
36. $-46 + 0$
37. $34 - (-18)$
38. $16 - 30$
39. $-6 \cdot (-15)$
40. $0(-8)$
41. $51 \div (-3)$
42. $-18 \div (-9)$

Basic Skills Find the unknown number.

43. $\underline{\quad} + 8 = 7$
44. $9 \times \underline{\quad} = 108$
45. $\underline{\quad} \div 12 = 6$

Test-Taking Practice

46. **Multiple Choice** Simplify the expression $\left(\frac{8^{-2}}{8}\right)^0$.

A. $\frac{1}{512}$ B. $\frac{1}{8}$ C. 1 D. 8

47. **Multiple Choice** Simplify the expression $\frac{-3x^{-4}}{x^2}$.

F. $\frac{-3}{x^6}$ G. $-3x^6$ H. $\frac{-3}{x^{-6}}$ I. $\frac{-3x}{x^6}$

LESSON 4.8

Scientific Notation

BEFORE

You multiplied numbers by powers of 10.

Now

You'll read and write numbers using scientific notation.

WHY?

So you can find the number of new \$1 bills printed, as in Ex. 39.

Word Watch

scientific notation, p. 205

In the Real World

Bubbles The brilliant colors observed in soap bubbles occur as a result of light reflecting from the inner and outer surfaces of the bubble. The thickness of a soap bubble is about 0.000004 meter. How can you use the powers of 10 to write 0.000004? You will see how to solve this problem in Example 1, part (a).

One way to write very small or very large numbers is to use *scientific notation*.



Using Scientific Notation

A number is written in **scientific notation** if it has the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer.

Standard form	Product form	Scientific notation
325,000	$3.25 \times 100,000$	3.25×10^5
0.0005	5×0.0001	5×10^{-4}

HELP with Solving

Powers of ten

$10^5 = 100,000$
$10^4 = 10,000$
$10^3 = 1,000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$
$10^{-1} = 0.1$
$10^{-2} = 0.01$
$10^{-3} = 0.001$
$10^{-4} = 0.0001$
$10^{-5} = 0.00001$

EXAMPLE 1 Writing Numbers in Scientific Notation

- a. The thickness of a soap bubble is about 0.0000004 meter.

Standard form	Product form	Scientific notation
0.0000004	4×0.000001	4×10^{-6}
Move decimal point 6 places to the right.		Exponent is -6 .

- b. There are over 300,000,000,000 stars in the Andromeda Galaxy.

Standard form	Product form	Scientific notation
300,000,000,000	$3 \times 100,000,000,000$	3×10^{11}
Move decimal point 11 places to the left.		Exponent is 11.

EXAMPLE 2 Writing Numbers in Standard Form

Scientific notation	Product form	Standard form
a. 7.2×10^5 Exponent is 5.	$7.2 \times 100,000$	720,000 Move decimal point 5 places to the right.
b. 4.65×10^{-7} Exponent is -7.	4.65×0.0000001	0.000000465 Move decimal point 7 places to the left.

Your turn now Write the number in scientific notation.

- | | | |
|------------|---------------|-------------------|
| 1. 4000 | 2. 7,300,000 | 3. 63,000,000,000 |
| 4. 0.00475 | 5. 0.00000526 | 6. 0.0000000082 |

Write the number in standard form.

- | | | |
|--------------------------|---------------------------|---------------------------|
| 7. 3.5×10^3 | 8. 2.48×10^6 | 9. 6×10^{11} |
| 10. 5.1×10^{-4} | 11. 9.16×10^{-2} | 12. 1.02×10^{-8} |

You can use the product of powers property to multiply two numbers written in scientific notation.

Watch Out!

When a number is in scientific notation, the factor c must be greater than or equal to 1 and less than 10. The number 28.35×10^{10} is not written in scientific notation because $28.35 > 10$.

EXAMPLE 3 Multiplying Numbers in Scientific Notation

Find the product $(4.5 \times 10^3) \times (6.3 \times 10^7)$.

Solution

$$\begin{aligned}
 &(4.5 \times 10^3) \times (6.3 \times 10^7) \\
 &= 4.5 \times 6.3 \times 10^3 \times 10^7 \\
 &= (4.5 \times 6.3) \times (10^3 \times 10^7) \\
 &= 28.35 \times 10^{10} \\
 &= 2.835 \times 10^1 \times 10^{10} \\
 &= 2.835 \times 10^{11}
 \end{aligned}$$

Commutative property of multiplication

Associative property of multiplication

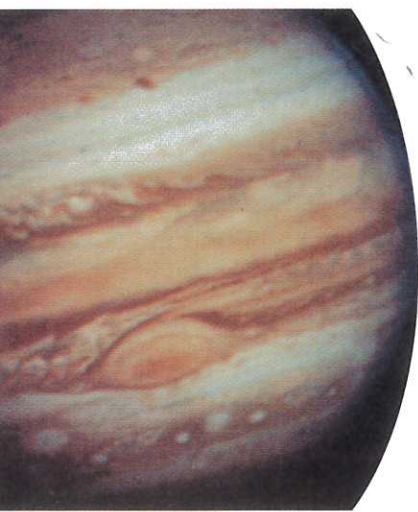
Product of powers property

Write 28.35 in scientific notation.

Product of powers property

Your turn now Write the product in scientific notation.

- | | |
|--|---|
| 13. $(1.25 \times 10^6) \times (7.6 \times 10^{12})$ | 14. $(8 \times 10^5) \times (5.65 \times 10^4)$ |
|--|---|



Getting Ready to Practice

Vocabulary Tell whether the number is expressed in scientific notation.

1. 9.32×10^5

2. 56.8×10^2

3. 7×10^{-4}

Write the number in scientific notation.

4. 89,200,000,000

5. 0.468

6. 0.0000671

Write the number in standard form.

7. 4.35×10^6

8. 5.72×10^{-3}

9. 9.62×10^7

10. **Guided Problem Solving** The mass of Earth is about 1.3×10^{25} pounds. The mass of Jupiter is about 4.2×10^{27} pounds. About how many times greater is Jupiter's mass than Earth's mass?

(1) Write the quotient of 4.2 and 1.3 as a decimal.

(2) Write the quotient of the powers of 10.

(3) Write the product of the quotients in scientific notation.

Practice and Problem Solving

HELP

with Homework

Example Exercises

- | | |
|---|-----------|
| 1 | 11-16, 27 |
| 2 | 17-22, 28 |
| 3 | 23-26 |



Online Resources
CLASSZONE.COM

- More Examples
- eTutorial Plus

Write the number in scientific notation.

11. 7900

12. 8,100,000,000

13. 2,130,000

14. 0.0312

15. 0.000000415

16. 0.0000000342

Write the number in standard form.

17. 8.71×10^{-2}

18. 6.35×10^{-6}

19. 1.76×10^{-9}

20. 4.13×10^9

21. 2.83×10^{12}

22. 3.61×10^7

Write the product in scientific notation.

23. $(3 \times 10^3) \times (2 \times 10^5)$

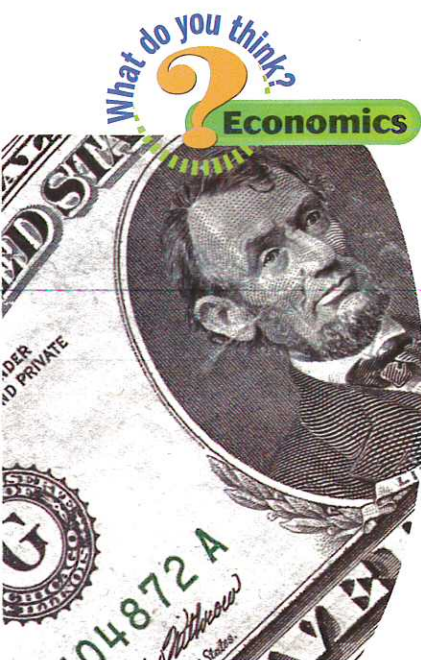
24. $(8 \times 10^6) \times (7 \times 10^4)$

25. $(7.8 \times 10^6) \times (8.4 \times 10^7)$

26. $(3.6 \times 10^8) \times (5.2 \times 10^5)$

27. **Well Water** In the United States, 15,000,000 households use private wells for their water supply. Write this number in scientific notation.

28. **State Parks** The United States has a total of 1.2916×10^7 acres of land reserved for state parks. Write this number in standard form.



■ U.S. Currency

The United States Bureau of Engraving and Printing prints about 4,440,000 new \$5 bills each day. What is the dollar value of these bills?

29. **Number Sense** Explain how you can tell whether a number is very small or very large when the number is written in scientific notation.

Copy and complete the statement with $<$, $>$, or $=$.

30. $6.92 \times 10^{11} \underline{\hspace{1cm}} 6.92 \times 10^{12}$

31. $3.67 \times 10^{-3} \underline{\hspace{1cm}} 3.76 \times 10^{-4}$

Find the product or quotient. Write your answer in scientific notation.

32. $(6.8 \times 10^{-2}) \times (3.9 \times 10^{-5})$

33. $(2.6 \times 10^7) \times (4.1 \times 10^{-3})$

34. $(7.6 \times 10^{-8}) \times (4.8 \times 10^{-6})$

35. $(5.4 \times 10^{-5}) \times (3.6 \times 10^{-9})$

36. $\frac{4.08 \times 10^6}{3.4 \times 10^2}$

37. $\frac{2.765 \times 10^{21}}{7.9 \times 10^9}$

38. $\frac{5.46 \times 10^{28}}{6.5 \times 10^{24}}$

39. **U.S. Currency** The United States Bureau of Engraving and Printing prints about 17 million new \$1 bills each day. About how many bills are printed in one week? in one year? Write your answers in scientific notation.

40. **Critical Thinking** Order the numbers from least to greatest.

3.75×10^8 37,500,000 3.57×10^9 5.37×10^7

41. **Science** The radius of a proton is about 1.2 Fermis. One Fermi is equal to 10^{-15} meter. How many centimeters is the radius of a proton? Write your answer in scientific notation.

42. **Challenge** Light travels 1.86×10^5 miles in 1 second. How far does light travel in one year?

Mixed Review

In Exercises 43–44, evaluate the expression. (Lessons 2.2, 2.3)

43. $-8 + 12 + (-16) + 18$

44. $34 - (-43) - (3 - 6)$

45. Simplify $\frac{6n}{9mn}$. (Lesson 4.3)

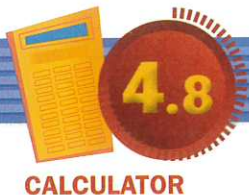
46. **Basic Skills** Find the amount of time that has elapsed from 10:46 A.M. to 3:13 P.M.

Test-Taking Practice

47. **Multiple Choice** In 2000, there were approximately 281,000,000 people in the United States. Which of the following is *not* another way of expressing the number 281,000,000?

A. 28.1 million B. 0.281 billion C. 28.1×10^7 D. 2.81×10^8

48. **Short Response** A space probe travels about 1.5×10^6 miles per day to its destination 21 million miles away. It has already traveled 9 million miles. About how many days of travel does it have left?



Technology Activity

Using Scientific Notation

GOAL Use a calculator to perform operations on numbers written in scientific notation.

Example

The Sun is about 1.5×10^8 kilometers from Earth, and Proxima Centauri is about 2.5×10^5 times farther from Earth than the Sun. How far is Proxima Centauri from Earth?

Solution

To find how far Proxima Centauri is from Earth, multiply the distance between the Sun and Earth by 2.5×10^5 .

Keystrokes

1.5 **EE** 8 **×** 2.5 **EE** 5 **=**

Display

3.75 $\times 10^{13}$

ANSWER Proxima Centauri is approximately 3.75×10^{13} kilometers from Earth.

Your turn now Use a calculator to evaluate the expression.

Write your answer in scientific notation.

1. $(3.19 \times 10^7) \times (8.5 \times 10^6)$ 2. $(6.7 \times 10^{-3}) \times (1.12 \times 10^{15})$

3. $(3.3 \times 10^{-3}) \times (4.8 \times 10^{-9})$ 4. $(7.1 \times 10^{-9}) \times (2.05 \times 10^6)$

5. $\frac{8.1 \times 10^{10}}{3.02 \times 10^3}$

6. $\frac{1.44 \times 10^{-15}}{1.6 \times 10}$

7. $\frac{2.8 \times 10^{-11}}{2.05 \times 10^{-4}}$

8. **Water** About 110 billion gallons of water flow through Lake Erie each day. How many gallons of water flow through Lake Erie in a week? in a year?

9. **Biology** The nucleus of a human cell is about 7×10^{-6} meter in diameter. A ribosome, another part of a cell, is about 3×10^{-8} meter in diameter. How many times larger is a nucleus than a ribosome?

Notebook Review



Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

least common denominator (LCD), p. 192

scientific notation, p. 205

Use Your Vocabulary

- Copy and complete: A number is written in ? if it has the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer.

4.5 Can you compare and order fractions?



EXAMPLE Compare $\frac{11}{18}$ and $\frac{3}{4}$.

The LCM of 18 and 4 is 36, so the least common denominator is 36.

$$\frac{11}{18} = \frac{11 \cdot 2}{18 \cdot 2} = \frac{22}{36}$$

$$\frac{3}{4} = \frac{3 \cdot 9}{4 \cdot 9} = \frac{27}{36}$$

ANSWER Because $\frac{22}{36} < \frac{27}{36}$, you can write $\frac{11}{18} < \frac{3}{4}$.



Copy and complete the statement with $<$, $>$, or $=$.

2. $\frac{2}{3} ? \frac{5}{9}$

3. $\frac{5}{12} ? \frac{1}{3}$

4. $\frac{4}{5} ? \frac{6}{7}$

5. $\frac{16}{24} ? \frac{20}{30}$

4.6 Can you use the rules of exponents?



EXAMPLE Multiply or divide. Write your answer as a power.

a. $x^7 \cdot x^8 = x^{7+8}$
 $= x^{15}$

b. $\frac{a^6}{a^3} = a^{6-3}$
 $= a^3$



Simplify. Write your answer as a power.

6. $n^4 \cdot n^9$

7. $y^6 \cdot y^{10}$

8. $\frac{x^7}{x^5}$

9. $\frac{c^{12}}{c^8}$

4.7 Can you use negative exponents?

Review

EXAMPLE Write $x^{-4} \cdot x^{-3}$ using only positive exponents.

$$x^{-4} \cdot x^{-3} = x^{-4 + (-3)} = x^{-7} = \frac{1}{x^7}$$



Simplify. Write the expression using only positive exponents.

10. $12a^{-5}$

11. $n^7 \cdot n^{-10}$

12. $\frac{m^{-6}}{m^5}$

13. $\frac{c^{-9}}{c^4}$

4.8 Can you write a number in scientific notation?

Review

EXAMPLE Write the number in scientific notation.

a. $980,000,000 = 9.8 \times 10^8$

b. $0.000012 = 1.2 \times 10^{-5}$



Write the number in scientific notation.

14. $34,600,000,000$

15. 0.0000009

16. 0.000000000502

Stop and Think

about Lessons 4.5–4.8



17. **Writing** Explain how to find the product $(5 \times 10^9) \times (4 \times 10^{15})$ without using a calculator.

Review Quiz 2

Copy and complete the statement with $<$, $>$, or $=$.

1. $\frac{2}{5} ? \frac{6}{15}$

2. $\frac{5}{6} ? \frac{4}{9}$

3. $\frac{9}{15} ? \frac{5}{9}$

4. $\frac{35}{40} ? \frac{21}{24}$

Order the numbers from least to greatest.

5. $\frac{2}{3}, \frac{5}{6}, \frac{1}{2}, \frac{5}{12}$

6. $1\frac{4}{7}, 1\frac{5}{14}, \frac{5}{4}, 1\frac{5}{8}$

Multiple or divide. Write your answer as a power using only positive exponents.

7. $b^2 \cdot b^4$

8. $c^5 \cdot c^{-2}$

9. $\frac{a^7}{a^2}$

10. $\frac{n^{-2}}{n^3}$

11. **Popcorn** People in the United States eat 1,120,000,000 pounds of popcorn a year. Write this number in scientific notation.

Chapter Review

Vocabulary

- | | |
|--------------------------------------|--|
| prime number, p. 169 | simplest form, p. 179 |
| composite number, p. 169 | equivalent fractions, p. 179 |
| prime factorization, p. 169 | multiple, p. 186 |
| factor tree, p. 169 | common multiple, p. 186 |
| monomial, p. 170 | least common multiple (LCM), p. 186 |
| common factor, p. 173 | least common denominator (LCD), p. 192 |
| greatest common factor (GCF), p. 173 | scientific notation, p. 205 |
| relatively prime, p. 174 | |

Vocabulary Review

- | | |
|---|--|
| 1. Describe the difference between the <i>greatest common factor</i> and the <i>least common multiple</i> of two numbers. | Copy and complete the statement. |
| 2. Give three examples of prime numbers greater than 20. | 5. A fraction is in <u>?</u> if its numerator and denominator have 1 as their GCF |
| 3. Give three examples of monomials. | 6. A(n) <u>?</u> is a whole number that has positive factors other than 1 and itself. |
| 4. Describe what it means for two numbers to be relatively prime. | 7. When you write a number as the product of prime numbers, you are writing its <u>?</u> . |
| | 8. Two fractions are <u>?</u> if they represent the same number. |

Review Questions

Write the prime factorization of the number. (Lesson 4.1)

- | | | | |
|-------|--------|---------|---------|
| 9. 54 | 10. 70 | 11. 150 | 12. 184 |
|-------|--------|---------|---------|

Factor the monomial. (Lesson 4.1)

- | | | | |
|--------------|--------------|----------------|----------------|
| 13. $19a^2b$ | 14. $28xy^3$ | 15. $56u^2v^2$ | 16. $80p^4q^3$ |
|--------------|--------------|----------------|----------------|

Find the greatest common factor of the numbers or monomials.
(Lesson 4.2)

- | | | |
|--------------------|--------------------|-------------------------|
| 17. 20, 40, 90 | 18. 56, 84, 196 | 19. 48, 60, 165 |
| 20. $2x, x^2, x^3$ | 21. $18xy^2, 81xy$ | 22. $54s^4t^4, 164st^3$ |

Review Questions

Write the fraction in simplest form. (Lesson 4.3)

23. $\frac{16}{48}$

24. $-\frac{38}{95}$

25. $-\frac{32}{102}$

26. $\frac{104}{39}$

27. $\frac{3bc}{9b}$

28. $-\frac{9abc}{12a}$

29. $\frac{20m}{5mn}$

30. $\frac{21bcd}{7bc}$

Find the least common multiple of the numbers or monomials.
(Lesson 4.4)

31. 15, 35

32. 180, 240

33. $5m^2n^4, 25mn^3$

34. $6p^2q^3r^4, 14pq^2r^3$

35. **Fountain** A fountain in an amusement park has special-effect devices called *shooters*. They shoot columns of water at different time intervals. One shooter goes off every 8 seconds while another goes off every 12 seconds. How long after the fountain is turned on will both shooters go off at the same time? (Lesson 4.4)



Copy and complete the statement with $<$, $>$, or $=$. (Lesson 4.5)

36. $\frac{79}{16} \text{ ? } \frac{35}{8}$

37. $6\frac{2}{3} \text{ ? } \frac{81}{12}$

38. $\frac{161}{9} \text{ ? } 17\frac{8}{9}$

39. $\frac{223}{15} \text{ ? } 14\frac{4}{5}$

40. **Calories** One serving of rice pilaf has 220 calories, including 35 calories from fat. One serving of soup has 70 calories, including 15 calories from fat. Write the calories from fat as a fraction of the total calories for each food. Which food has a greater fraction of calories from fat? (Lesson 4.5)

Simplify the expression. Write your answer as a power. (Lesson 4.6)

41. $8 \cdot 8^3$

42. $2^2 \cdot 2^5$

43. $7^9 \div 7^7$

44. $\frac{5^{10}}{5^7}$

Simplify. Write the expression using only positive exponents. (Lesson 4.7)

45. $7x^{-4}$

46. $a^{-6} \cdot a^4$

47. $\frac{8w^{-6}}{24w^2}$

48. $\frac{16r^{-2}}{4r^3}$

49. Write 6.58×10^{-4} in standard form. (Lesson 4.8)

50. Write 78,900,000,000 in scientific notation. (Lesson 4.8)

51. **Niagara Falls** In tourist season, the water at Niagara Falls flows at 100,000 cubic feet per second during the day. How fast does it flow per minute? per hour? Write your answers in scientific notation. (Lesson 4.8)

Chapter Test

Write the prime factorization of the number.

1. 49

2. 68

3. 95

4. 112

Find the greatest common factor of the monomials.

5. $3pq, 12pq$

6. $12a^2, 18ab$

7. $2z^3, 3z^2$

8. $14r^2, 42r$

Find the least common multiple of the numbers or monomials.

9. 4, 16, 32

10. 18, 24, 36

11. $5x^2y, 21xy^3$

12. $54pq^2, 63p^3q^3$

Copy and complete the statement with $<$, $>$, or $=$.

13. $\frac{11}{12} ? \frac{41}{48}$

14. $4\frac{3}{6} ? \frac{9}{2}$

15. $8\frac{7}{16} ? \frac{17}{2}$

16. **Cake** Three equal-sized round layer cakes were served at a party. Each cake was cut into a different number of equal-sized slices. After the guests left, $\frac{1}{8}$ of the yellow cake, $\frac{3}{16}$ of the chocolate cake, and $\frac{1}{6}$ of the carrot cake remained. Which type of cake had the least amount left over? Which had the most? Explain your reasoning.



Simplify the expression. Write your answer as a power.

17. $m^8 \cdot m^3$

18. $6^2 \cdot 6^6$

19. $\frac{n^{16}}{n^{10}}$

Simplify. Write the expression using only positive exponents.

20. $5x^{-3}$

21. $c^{-1} \cdot c^{-7}$

22. $\frac{-4u^{-9}}{u^3}$

23. $\frac{16a^2b^5}{8a^4b}$

24. **Science** Scientists have created a microfabric using molded plastic.

Its narrowest links are $\frac{1}{1,000,000}$ meter. Write this fraction as a power of ten.

Write the product in scientific notation.

25. $(6 \times 10^5) \times (5 \times 10^7)$

26. $(8.1 \times 10^4) \times (9.2 \times 10^8)$

27. $(4.2 \times 10^{-5}) \times (6 \times 10^{-2})$



Chapter Standardized Test

Test-Taking Strategy Be careful about choosing an answer that seems obvious. Carefully read the problem and all the choices before answering.

Multiple Choice

- Which number is a prime number?
A. 51 B. 67 C. 82 D. 93
- What is the greatest common factor of 420 and 385?
F. 5 G. 15 H. 35 I. 4620
- Which fraction is written in simplest form?
A. $\frac{3}{16}$ B. $\frac{4}{10}$ C. $\frac{9}{21}$ D. $\frac{15}{33}$
- Two toy cars begin at the starting line of a circular track at the same time. Car A goes around the track every 20 seconds. Car B goes around the track every 8 seconds. In how many seconds will the two cars reach the starting line at the same time?
F. 4 seconds G. 24 seconds
H. 40 seconds I. 60 seconds
- Which list is *not* in order from least to greatest?
A. $\frac{1}{4}, \frac{3}{8}, \frac{7}{12}, \frac{2}{3}$
B. $\frac{1}{2}, \frac{3}{4}, \frac{13}{16}, \frac{7}{8}$
C. $1\frac{5}{18}, 1\frac{7}{9}, \frac{17}{12}, \frac{11}{6}$
D. $2\frac{4}{21}, 2\frac{5}{14}, \frac{18}{7}, \frac{17}{6}$
- Which expression is *not* equal to 5^4 ?
F. $5^3 \cdot 5$ G. $5^2 \cdot 5^2$
H. $\frac{5^8}{5^4}$ I. $\frac{5^8}{5^2}$

- Which number is equal to $\frac{2^9}{2^3}$?
A. 8 B. 64 C. 520 D. 4096
- Write $\frac{-5x^{-6}}{x^3}$ using only positive exponents.
F. $\frac{-5}{x^9}$ G. $\frac{1}{5x^9}$ H. $-5x^6$ I. $30x^3$
- Simplify $(5 \times 10^{-7}) \times (3.6 \times 10^4)$.
A. 1.8×10^{-4} B. 1.8×10^{-3}
C. 1.8×10^{-2} D. 18×10^{-4}

Short Response

- Planting Trees** A conservation group wants to plant 48 trees in a rectangular arrangement so that each row has the same number of trees. How many trees can be planted in each row? List all possibilities. Of the possible arrangements, which one is closest to having a length three times its width?

Extended Response

- History** The Orb of 1661 is a gold sphere set with 365 diamonds, 363 pearls, 18 rubies, 9 emeralds, 9 sapphires, and 1 amethyst. What is the total number of jewels? What fraction of jewels are rubies? What fraction are emeralds? Write each fraction in simplest form. Jane estimates that about half of the jewels in the Orb are diamonds. Do you agree with this estimate? Explain.

Rational Number Operations

BEFORE

In previous chapters you've...

- Added, subtracted, multiplied, and divided integers
- Interpreted tables and graphs

Now

In Chapter 5 you'll study...

- Performing operations on fractions, mixed numbers, and decimals
- Rewriting fractions and decimals
- Describing data sets using mean, median, mode, and range

WHY?

So you can solve real-world problems about...

- snakes, p. 220
- sledding, p. 226
- rafting, p. 251
- deep sea jellies, p. 257



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Chapter Warm-Up Games

Review skills you need for this chapter in these quick games.

The game board is a grid of fractions. The starting point is a box with the fraction $\frac{2}{3}$. The goal is to reach the top of the cliff by moving up, selecting a handhold in each row. The handholds are represented by fractions in a grid. The fractions in the grid are:

$\frac{7}{32}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{3}{11}$
$\frac{7}{17}$	$\frac{4}{9}$	$\frac{4}{15}$	$\frac{11}{25}$
$\frac{6}{11}$	$\frac{13}{21}$	$\frac{4}{7}$	$\frac{3}{8}$
$\frac{1}{2}$	$\frac{5}{8}$	$\frac{7}{11}$	$\frac{9}{14}$
$\frac{3}{4}$	$\frac{5}{6}$	$\frac{3}{5}$	$\frac{5}{7}$

The title "Scale the Cliff" is written vertically on the right side of the board. An image of a person climbing a cliff is also shown.



Key Skill:
Comparing fractions

Find the handholds you can use to scale the cliff.

- Start at $\frac{2}{3}$ and move up, selecting a handhold in each row.
- The value of each handhold must be less than the value of the handhold below it.