

Integer Operations

BEFORE

In previous chapters you've...

- Performed operations on whole numbers
- · Evaluated expressions

Now

In Chapter 2 you'll study...

- Operations on integers
- Using properties to evaluate expressions
- Identifying and plotting points in the coordinate plane

WHY?

So you can solve real-world problems about...

- space shuttles, p. 56
- · dinosaurs, p. 66
- diving, p. 70
- murals, p. 89

Internet Preview

CLASSZONE.COM

- eEdition Plus Online
- eWorkbook Plus Online
- eTutorial Plus Online
- State Test Practice
- More Examples

Chapter Warm-Up Game

Review skills you need for this chapter in this quick game. Work with a partner.

Key Skill:

Multiplying whole numbers

FOUR IN A ROW

MATERIALS

- 2 Answer Cards
- 24 Expression Cards
- 24 Markers







PREPARE Fill in your Answer Card with 16 of the 24 answers given below. Place the Expression Cards face down in a pile. On each turn follow the steps on the next page.

168	196	240	315	338	342
352	361	405	414	418	441
516	522	529	595	720	792
832	851	918	961	975	1020

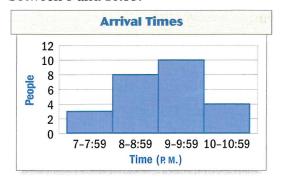


Chapter Standardized Test

Test-Taking Strategy Most standardized tests are based on concepts and skills taught in school. The best way to prepare is to keep up with your daily studies.

Multiple Choice

1. The histogram shows the times 25 people arrived at a party. How many people arrived between 9 and 10:59?



- **A.** 4
- **B.** 14
- **C.** 21
- **D.** 25
- **2.** In what order should the operations be performed in the expression $3 + 7 \times 4 \div 2 - 6$?

$$\mathbf{F}_{\bullet} \times . - . \div . +$$

F.
$$\times$$
, -, \div , + **G.** +, \times , \div , -

$$H. \times, \div, +, -$$

$$1. +, -, \times, \div$$

3. Which expression has a value of 20?

A.
$$15 + 5 \times 4 \div 2 - 1$$

B.
$$(15 + 5) \times (4 \div 2 - 1)$$

C.
$$(15 + 5) \times 4 \div 2 - 1$$

D.
$$(15+5) \times 4 \div (2-1)$$

- **4.** What is the correct value of the expression $(11-9)^4+6\times 3?$
 - **F.** 26
- **G.** 34
- 1.66
- **5.** Which statement is true?

A.
$$2^6 < 6^2$$

B.
$$4^7 < 7^4$$

C.
$$1^9 > 9^{1}$$

D.
$$3^5 > 5^3$$

6. Which equation represents this statement: The quotient of twenty and a number is five?

F.
$$20 \div r = 5$$

G.
$$t \div 20 = 5$$

H.
$$5 \div p = 20$$

1.
$$20g = 5$$

7. What is the solution of the equation in Exercise 6?

A.
$$\frac{1}{4}$$

B. 2

C. 4

D. 100

8. If a = 4 and b = 9, which equation is true?

F.
$$a = 5 + b$$

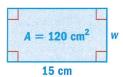
G.
$$ab = 13$$

H.
$$b \div 3 = a$$

$$1.2a + b = 17$$

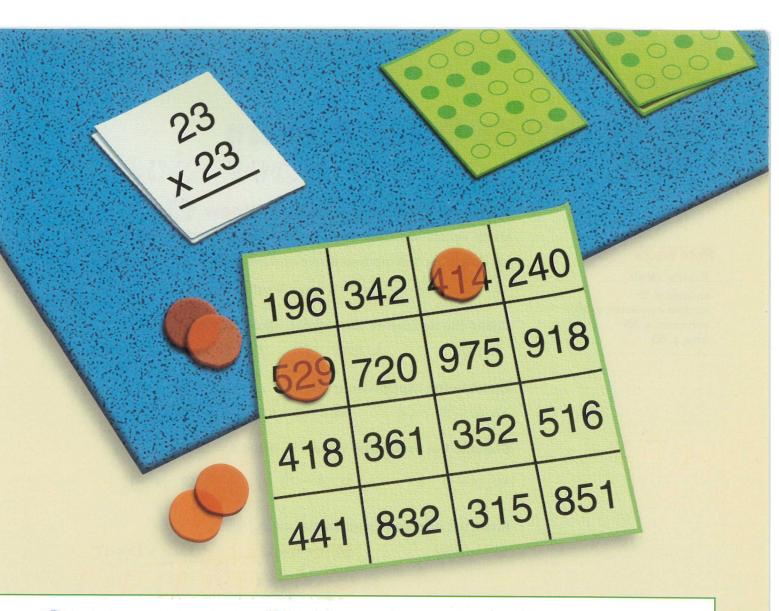
Short Response

9. The area of the rectangle is 120 square centimeters. Write an equation you can use to find the width w. Then solve the equation for w.



Extended Response

10. Your car's fuel gauge is broken. The car can go 22 miles on one gallon of gasoline. You start a trip with 13 gallons of gasoline. If you want to always have at least a gallon in the tank, what is the farthest you should drive before stopping for more gasoline? Explain.







2 LOOK for the answer on your Answer Card. If you find it, place a marker over the answer.



HOW TO WIN Mark 4 answers in a row across, up and down, or diagonally.



Stop and Think

- 1. How many ways can you get 4 answers in a row on your card?
- 2. Critical Thinking How many squares can you mark without winning?



Getting Ready to Learn

Word Watch

Review Words

variable, p. 15 variable expression, p. 15 perimeter, p. 33 area, p. 33

Review What You Need to Know <



Using Vocabulary Copy and complete using a review word.

- **1.** A symbol that represents one or more numbers is called a(n) ?.
- **2.** The surface covered by a figure is called the ?.

Round the decimal to the nearest whole number. (p. 705)

- **3.** 10.61
- **4.** 134.7
- **5.** 0.25
- **6.** 12.86

Evaluate the expression. (p. 10)

9.
$$6 + 34 \div 2$$

Evaluate the expression when s = 4 and t = 16. (p. 15)

10.
$$(t-9)+s$$

11.
$$s(t-5)$$

12.
$$\frac{1}{4}t - 4$$

Solve the equation using mental math. (p. 28)

13.
$$3x = 39$$

14.
$$x - 6 = 12$$

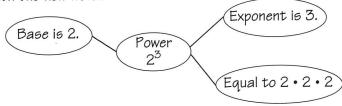
15.
$$x + 13 = 17$$



You should include material that appears on a notebook like this in your own notes.

Know How to Take Notes

Including Vocabulary Notes When you write down new vocabulary words, you should also write examples of how they are used. Label the examples with the new words.



 5^2 is read "five squared."

5³ is read "five cubed."

54 is read "five to the fourth power."

As you work through Chapter 2, label examples of new vocabulary in your notes.



Integers and Absolute Value

BEFORE

Now

WHY?

You studied whole numbers.

You'll study integers.

So you can order lake elevations, as in Ex. 24.

Word Watch

integer, p. 53 negative integer, p. 53 positive integer, p. 53 absolute value, p. 54 opposite, p. 54

In the Real World

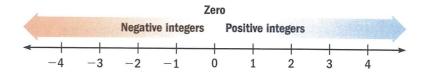
Geography The Global Positioning System (GPS) can be used to determine elevations. The table shows the minimum elevations of several countries. Which country in the table has the lowest elevation?

Each number in the table is an integer. The following numbers are **integers**.

Country	Elevation (m		
United States	-86		
Canada	0		
China	-154		
Bolivia	90		
Czech Republic	115		

$$\ldots$$
, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots

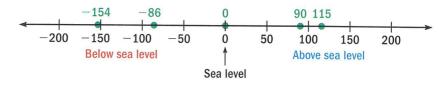
Negative integers are less than 0. They lie to the left of 0 on a number line. **Positive integers** are greater than 0. They lie to the right of 0 on a number line. Zero is neither positive nor negative. When you use a number line to compare numbers, numbers increase as you move to the right.



Global Positioning System (GPS) satellite

Graphing and Ordering Integers

To find which country in the table above has the lowest elevation, graph each integer on a number line.

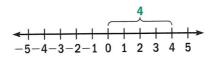


ANSWER China has the lowest elevation, at -154 meters.

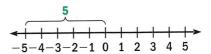
Your turn now Order the integers from least to greatest.

1.
$$-7$$
, 2, -1 , 0, -2

The **absolute value** of a number is the distance between the number and zero on a number line. The absolute value of a number n is written as |n|. The absolute value of 0 is 0.



The distance between 4 and 0 is 4. So, |4| = 4.



The distance between -5 and 0 is 5. So, |-5| = 5.



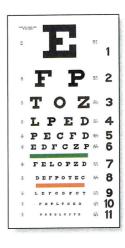
EXAMPLE 2 Finding Absolute Value

Eyeglasses An eyeglass prescription is given as a positive or negative number. A prescription of a person who is farsighted is positive. A prescription of a person who is nearsighted is negative. The greater the absolute value, the stronger the prescription. Which prescription is stronger, -3 or 2?

Solution

$$|-3| = 3$$
 and $|2| = 2$.

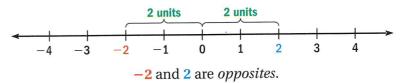
ANSWER The prescription of -3 is stronger because 3 > 2.





The integer "-2" can be read "negative 2" or "the opposite of 2."

Two numbers are **opposites** if they have the same absolute value but different signs. Opposites are the same distance from 0 on a number line and are on opposite sides of 0. The opposite of 0 is 0.



EXAMPLE 3 Finding Opposites

Write the opposite of the integer.

a. 5 The opposite of 5 is -5.

b. -12 The opposite of -12 is 12.

c. |-9| Because |-9| = 9, the opposite of |-9| is -9.

Your turn now

Write the opposite and the absolute value of the integer.

4. -16

5. 140

6. 1

7. | -55 |



Getting Ready to Practice

1. Vocabulary Copy and complete: Two integers are ? if they have the same absolute value but different signs.

Use a number line to order the integers from least to greatest.

Write the opposite and the absolute value of the integer.

6.
$$-740$$

8. Find the Error A student was asked to order the integers 3, 1, 0, -9, -2, and 5 from least to greatest. Describe and correct the error in the solution.





Example Exercises

1 9-18

2 19-2

3

19-22 19-22



- · More Examples
- · eTutorial Plus

Practice and Problem Solving

Copy and complete the statement with < or >.

Use a number line to order the integers from least to greatest.

Match the integer expression with the verbal expression.

A. the opposite of negative seven

20.
$$|-7|$$

B. the opposite of the absolute value of seven

c. the absolute value of negative seven

22.
$$-(-7)$$

D. the opposite of the absolute value of negative seven

In Exercises 23 and 24, use the table showing elevations of lakes.

- **23. Compare** Which lake is at a lower elevation, Gieselmann Lake or Silver Lake?
- **24.** Arrange the lake elevations in order from least to greatest.

Name	Elevation (ft)
Jones Lake	-30
Silver Lake	90
Gieselmann Lake	-162
Seneca Lake	445
Craigs Pond	0



The opposite of an opposite is the original number. For example, -(-16) = 16.

Simplify the expression.

28.
$$-(-5)$$

29.
$$-(-81)$$

30.
$$-|-17|$$

31.
$$- | -3 |$$

26.
$$-|9|$$
 27. $-|29|$ **28.** $-(-5)$ **30.** $-|-17|$ **31.** $-|-3|$ **32.** $-(-(-4))$

Copy and complete the statement with <, >, or =.

34.
$$|-6|$$
 ? $-|6|$

34.
$$|-6|$$
 ? $-|6|$ **35.** $-|-9|$? $-(-9)$

- 36. Critical Thinking What numbers have opposites that are the same as their absolute values? What numbers have opposites that are different from their absolute values?
- **37.** Launch Countdown Put the following activities for a shuttle launch in the order that they occur. "T-5 minutes" means 5 minutes before liftoff.

T-5 minutes	Pilot starts auxiliary power units.		
T+7 seconds	Shuttle clears launch tower, and control		
	switches to the Mission Control Center.		
T-2 hours, 55 minutes	Flight crew departs for launch pad.		
T-6 seconds	Main engine starts.		
T-0	Liftoff.		

Challenge Order the numbers from least to greatest.

Mixed Review (1)

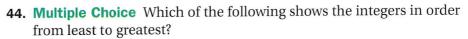
Evaluate the expression when a = 8 and b = 2. (Lesson 1.3)

41.
$$\frac{a}{b} + 15$$

43. Patty needs to read a 238 page book in 6 days. By the end of the first day she has read 68 pages. How many pages does she need to read each day to finish the book on time? (Lesson 1.7)

Test-Taking Practice





B.
$$-1$$
, -12 , -34 , -6

D.
$$-34, -6, -12, -1$$

F.
$$-7258$$
 G. $-(-7258)$ **H.** $|-7258|$ **I.** $|7258|$

INTERNET

State Test Practice

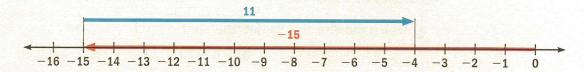
- Model integer addition on a number line.
- · pencil paper

Adding Integers

You can model addition of integers by using a number line.

Find the sum -15 + 11.

- Draw a number line, place a pencil at 0, and move 15 units to the left to show -15.
- Move 11 units to the right to show the addition of 11.

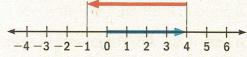


The final position is -4. So, -15 + 11 = -4.

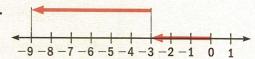
Your turn now Write an addition expression to represent the figure.

Then evaluate the expression.

1.



2.



Use a number line to find the sum.

3.
$$-7 + (-14)$$

4.
$$20 + (-50)$$

5.
$$-10 + 65$$

6.
$$-7 + (-33)$$

7.
$$41 + (-25)$$

8.
$$-23 + 52$$

9.
$$-18 + (-34)$$
 10. $35 + (-37)$

10.
$$35 + (-37)$$

Stop and Think

- **11.** The sum of two positive integers is always positive. What is the sign of the sum of two negative integers? Use a number line to explain.
- 12. Critical Thinking How can you predict the sign of the sum of a positive and a negative integer before you add the numbers?
- **13.** Writing Write the steps you use to evaluate 25 + (-13) + 5 + (-20). Then evaluate the expression.



Adding Integers

BEFORE

Now

WHY?

You added whole numbers.

You'll add integers.

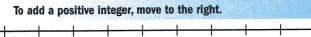
So you can find a miniature golf score, as in Ex. 43.



Review Words

integer, p. 53 absolute value, p. 54 sum, p. 709

You can use a number line to add integers.



To add a negative integer, move to the left.

EXAMPLE 1 Adding Integers Using a Number Line

Use a number line to find the sum.

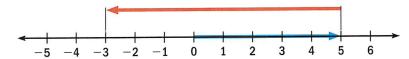
a.
$$5 + (-8)$$

b.
$$-6 + 10$$

c.
$$-4 + (-3)$$

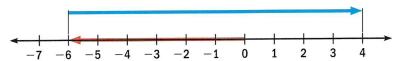
Solution

a. Start at 0, move 5 units to the right. Then move 8 units to the left.



ANSWER The final position is -3. So, 5 + (-8) = -3.

b. Start at 0, move 6 units to the left. Then move 10 units to the right.



ANSWER The final position is 4. So, -6 + 10 = 4.

c. Start at 0, move 4 units to the left. Then move 3 units to the left.



ANSWER The final position is -7. So, -4 + (-3) = -7.

Your turn now

Use a number line to find the sum.

1.
$$12 + (-5)$$

2.
$$-8+4$$

3.
$$-1 + (-6)$$

4.
$$2 + (-2)$$

Using a Rule You can add integers without using a number line by following these rules.

Same sign Add the absolute values and use the common sign.

Different signs Subtract the lesser absolute value from the greater absolute value. Use the sign of the number with the

absolute value. Use the sign of the number with the greater absolute value.

EXAMPLE 2 Adding Integers

Find the sum -12 + 4.

Different signs, so subtract
$$|4|$$
 from $|-12|$.

Use sign of number with greater absolute value.

✓ **Check** Use a number line to find the sum.





Additive Identity Property

Words The sum of an integer and zero is the integer.

Numbers
$$5 + 0 = 5$$
 Algebra $a + 0 = a$ $-3 + 0 = -3$

EXAMPLE 3 Adding More Than Two Integers

a. Use the left to right rule of order of operations to find the sum.

$$-84 + 0 + (-124) = -84 + (-124)$$
 Additive identity property
$$= -208$$
 Same sign, so sum has common sign.

b. Use the left to right rule of order of operations to find the sum.

$$-46 + (-53) + 63 = -99 + 63$$
 Same sign, so sum has common sign.
$$= -36$$
 Use sign of number with greater absolute value.



EXAMPLE 4 Adding More Than Two Integers



School Fair Your class has a fair to raise money for a field trip. The table shows the incomes and expenses for the fair. How much money was raised?

Games	\$750
Display tables	\$625
Donations	\$36
Advertising	-\$16
Decorations	-\$60
Game rentals	-\$500

Solution

First, add the **positive integers**, and then add the **negative integers**.

$$750 + 625 + 36 + (-16) + (-60) + (-500) = 1411 + (-576)$$

= 835

ANSWER Your class raised \$835.

Your turn now Find the sum.

5.
$$-20 + (-15)$$

6.
$$18 + 0 + (-54)$$

7.
$$300 + 111 + (-44) + (-256)$$

8.
$$-230 + (-512) + 178 + 94$$

Exercises

More Practice, p. 728



Getting Ready to Practice

1. Vocabulary Copy and complete: To add two integers with the same sign, add the ? and use the common sign.

Use a number line to find the sum.

2.
$$-6 + 8$$

3.
$$-3 + (-9)$$

4.
$$5 + (-7)$$

5.
$$-4+4$$

Find the sum.

6.
$$42 + (-23)$$

7.
$$-32 + 0$$

8.
$$-51 + (-67)$$

9.
$$-19 + 19 + (-34)$$
 10. $-12 + 9 + (-5)$ **11.** $20 + (-15) + (-22)$

10.
$$-12 + 9 + (-5)$$

11.
$$20 + (-15) + (-22)$$

12. Checking You record withdrawals and deposits in your checkbook. The starting balance is \$125. The first withdrawal is \$25. The second withdrawal is \$13. The first deposit is \$35. The second deposit is \$50. The third withdrawal is \$68. What is the final balance?



Example Exercises

1	13-16
THE PERSON NAMED IN	

- 17-26 2
- 3 18-26, 34-37
- 34-37



- · More Examples
- · eTutorial Plus

Practice and Problem Solving

Use a number line to find the sum.

13.
$$-2 + (-1)$$

14.
$$-10 + (-9)$$

16.
$$7 + (-5)$$

17. Find the Error Describe and correct the error in the solution.

Find the sum of
$$-8$$
 and 5 .

$$-8 + 5 = -13$$



Find the sum.

18.
$$-63 + (-49)$$

19.
$$-93 + (-16)$$

20.
$$0 + (-25)$$

21.
$$-82 + 0$$

23.
$$-57 + 31 + 27 + 11$$

24.
$$-42 + (-65) + 78$$
 25. $-87 + 48 + 36$

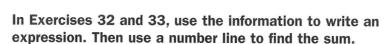
25.
$$-87 + 48 + 36$$

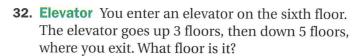
26.
$$-81 + (-75) + (-65)$$

Critical Thinking Copy and complete the statement using always, sometimes, or never.

- **27.** The sum of two negative integers is ? negative.
- **28.** The sum of two positive integers is ? negative.
- **29.** The sum of a positive integer and a negative integer is ? negative.
- **30.** The sum of an integer and zero is ? zero.

31. Writing Describe a situation where you would need to add positive and negative integers.





33. Mexico The influential period of the *Olmec* culture in Mexico lasted approximately 800 years. It started about 1200 B.C. About what year did this period end?



Jade sculpture from the Olmec culture in Veracruz. Mexico

Find the sum.

34.
$$42 + 36 + (-16) + 0 + (-84)$$

35.
$$(-17) + (-63) + 91 + 79$$

36.
$$174 + (-196) + 245 + (-210)$$

Algebra Evaluate x + (-478) for the value of x.

38.
$$x = 806$$

39.
$$x = -729$$

40
$$r = |-3/9|$$

40.
$$x = \begin{bmatrix} -349 \end{bmatrix}$$
 41. $x = -\begin{bmatrix} -521 \end{bmatrix}$

42. Making Connections The sum of a number and its *additive inverse* is 0. For example, 5 + (-5) = 0, so 5 and -5 are additive inverses. Give another example of additive inverses. What vocabulary word from this chapter is another name for additive inverses?



43. Miniature Golf In miniature golf, *par* is the number of strokes considered necessary to get a ball in the hole. The score for each hole is the number of strokes above or below par. Find the total score by adding the scores for each hole. Is Jill's score above par, under par, or at par?

HOLE	1	2	3	4	5	6	7	8	9	OUT
PAR	4	5	3	3	5	4	3	5	3	35
Jill	0	+1	-2	-1			+2	0	-1	
11		-			STATE OF THE PARTY					

Chemistry In Exercises 44–46, use the information below. Find the sum of the charges. Tell whether the atom is an ion.

- A proton has a charge of +1.
- An electron has a charge of -1.
- An atom is an ion if it has a positive or negative charge.
- **44.** Sodium: 11 protons, 10 electrons
- **45.** Chlorine: 17 protons, 17 electrons
- **46.** Oxide: 8 protons, 10 electrons

Mental Math Solve the equation using mental math.

47.
$$-3 + k = 2$$

48.
$$-6 = x + (-9)$$

49.
$$-7 = 12 + i$$

50. Challenge Does |x + y| = |x| + |y| if x and y are both positive? What if x and y are both negative? What if x is positive and y is negative? Explain.

Mixed Review 😂

Evaluate the expression. (Lesson 1.4)

51.
$$5^3 + 21 \div 7 - 6$$

52.
$$6^2 \cdot (2+4) \div 18$$

51.
$$5^3 + 21 \div 7 - 6$$
 52. $6^2 \cdot (2+4) \div 18$ **53.** $(12-4) \cdot (9-1)^2$

Order the integers from least to greatest. (Lesson 2.1)

Test-Taking Practice





57. Multiple Choice Evaluate
$$-498 + (-512) + 573 + (-645)$$
.

G.
$$-1082$$



Subtracting Integers

BEFORE

Now

WHY?

You added integers.

You'll subtract integers.

So you can find the length of dinosaur periods, as in Ex. 41.



Review Words

integer, p. 53 opposite, p. 54 difference, p. 709

Activity

You can use patterns and mental math to discover a rule for subtracting integers.

- Copy the table. In the second column, write the answer to the subtraction problem. Use a pattern to find the differences involving negative integers.
- (2 In the third column, complete the addition problem so the sum is equal to the number in the difference column.

Subtraction problem	Difference	Addition problem
3 – 3	0	3 + -3
3 – 2	?	3 + <u>?</u>
3 – 1	?	3 + <u>?</u>
3 - 0	?	3 + <u>?</u>
3 - (-1)	?	3 + <u>?</u>
3 - (-2)	?	3 + <u>?</u>
3 - (-3)	?	3 + <u>?</u>

- (3 How is the second number in the addition problems related to the second number in the subtraction problems?
- 4 Describe how to use addition to subtract integers.

In the activity above, you saw that when you subtract integers you can write the expression as an addition expression and then use the rules for adding integers.



Subtracting Integers

Words To subtract an integer, add its opposite.

Numbers
$$3-7=3+(-7)=-4$$
 Algebra $a-b=a+(-b)$
 $2-(-6)=2+6=8$ $a-(-b)=a+b$

EXAMPLE 1 Subtracting Integers

a.
$$-56 - (-9) = -56 + 9$$

Add the opposite of -9.

$$= -47$$

Add.

b.
$$-14 - 21 = -14 + (-21)$$

Add the opposite of 21.

$$= -35$$

Add.

Your turn now Find the difference.

$$2. -16 - 8$$

3.
$$38 - (-27)$$

2.
$$-16-8$$
 3. $38-(-27)$ **4.** $-76-(-109)$

EXAMPLE 2 Evaluating a Variable Expression

Evaluate 15 - a - b when a = 24 and b = -36.

Solution

$$15 - a - b = 15 - 24 - (-36)$$

Substitute 24 for a and -36 for b.

$$= 15 + (-24) - (-36)$$

Add the opposite of 24.

$$= -9 - (-36)$$

Add 15 and -24.

$$= -9 + 36$$

Add the opposite of -36.

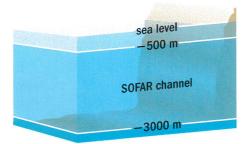
$$= 27$$

Add.



EXAMPLE 3 Using Integer Subtraction

SOFAR The SOFAR (SOund Fixing And Ranging) channel is a layer of water in the oceans that allows sounds to travel extremely long distances. Use the diagram to find the vertical height of the SOFAR channel.



SOFAR

Humpback whales use the SOFAR channel to communicate with other whales. The speed of sound in water at 68°F is 1482 meters per second. The speed of sound in air at 68°F is 344 meters per second. How much faster does sound travel in water?

Solution

The vertical height is the difference of the upper and lower elevations.

$$Vertical\ height = -500 - (-3000)$$

$$= -500 + 3000$$

$$= 2500$$

Write subtraction statement.

Add the opposite of -3000.

Add.

ANSWER The vertical height of the SOFAR channel is 2500 meters.

64



Getting Ready to Practice

Vocabulary Translate the verbal phrase into a numerical expression.

- **1.** The difference of negative two and six
- 2. The difference of the opposite of five and the opposite of three

Find the difference.

4.
$$6 - (-16)$$

4.
$$6 - (-16)$$
 5. $-11 - (-7)$ **6.** $-9 - 10$

6.
$$-9 - 10$$

Evaluate the expression when x = 15 and y = -8.

7.
$$5 - x$$

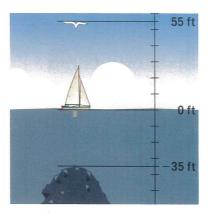
8.
$$-9 - y$$

9.
$$y - x$$

10.
$$x - y$$

- **11. Guided Problem Solving** Use the diagram to find the distances between the bird and the boat, the boat and the reef, and the bird and the reef.
 - (1 Identify which elevation is greater for each situation.
 - (2 Subtract the lower elevation from the greater elevation.
 - (3 Answer the original question by completing each statement.

The bird is ? feet above the boat. The boat is ? feet above the reef. The bird is ? feet above the reef.





Example Exercises

1 12-20

2 22-24

3 21, 26-28



- More Examples
- · eTutorial Plus

Practice and Problem Solving

Find the difference.

12.
$$-13 - 12$$

14.
$$11 - (-6)$$

16.
$$-18 - (-12)$$

20.
$$-39 - (-13)$$

21. Game Show A game show contestant has -400 points. He answers a question incorrectly and loses 600 points. What is his total score?

Evaluate the expression when c = -5 and d = 10.

24.
$$c - d - 8 - 4$$

25. Critical Thinking Explain how you can find the distance between the points on the number line using subtraction.



Temperatures In Exercises 26-28, use the table. It shows the coldest temperatures ever recorded for four states.

State	Temperature		
Alaska	-80°F		
Colorado	-61°F		
Kentucky	-37°F		
Mississippi	-19°F		

- 26. How much colder is Alaska's coldest temperature than Kentucky's?
- 27. How much colder is Colorado's coldest temperature than Mississippi's?
- **28.** Which two states have the greatest difference of coldest temperatures? Which two states have the least difference of coldest temperatures?

Evaluate the expression.

32.
$$-5 - (-5) - (-5)$$
 33. $8 - 2 - 6 - 10$

34.
$$-4 - 7 + (-9) - 1$$

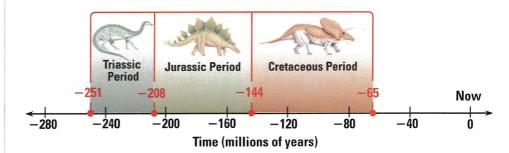
35.
$$-1 + (-8) - 9$$

37.
$$3 - (-7) - (-2)$$

38.
$$15 + (-29) - (-72)$$
 39. $-52 - (-18) - 37$ **40.** $91 + (-40) - 34$

40.
$$91 + (-40) - 34$$

41. Dinosaurs The table shows the ranges of three dinosaur periods during the Mesozoic Era. Calculate how long each of the periods lasted.



Dinosaurs

Scientists believe that if a Tyrannosaurus Rex was able to run more than 25 miles per hour it would have to have had more than 80% of its body mass in its legs. If a Tyrannosaurus Rex weighed 13,000 pounds, this would mean the legs weighed 10,400 pounds. How much would the rest of its body have weighed?

Science

Evaluate the expression when a = -9, b = 18, and c = -4.

42.
$$a + c - 10$$

43.
$$14 - a - b$$

44.
$$a - b - c$$

45.
$$c + a - b$$

Challenge Copy and complete the statement using always, sometimes, or never.

- **46.** A negative number minus a positive number is ? negative.
- **47.** A positive number minus a negative number is ? negative.

48. Weather To collect data on winter storms developing in the north Pacific Ocean, a cylinder that measures wind speed, humidity, and temperature at different elevations is dropped from a plane. If the plane is 40,000 feet above sea level and the cylinder drops to the bottom of the Pacific Ocean, 13,000 feet below sea level, how far does the cylinder fall?

Mixed Review 😂



Solve the equation using mental math. (Lesson 1.5)

49.
$$v + 5 = 13$$

50.
$$7w = 42$$

51.
$$12 - x = 9$$

Find the sum. (Lesson 2.2)

52.
$$-8 + 17$$

53.
$$11 + (-9)$$

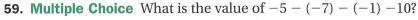
54.
$$-22 + (-6)$$

Basic Skills Find the difference.

Test-Taking Practice







60. Multiple Choice Evaluate the expression
$$-c + 2a - b$$
 when $a = 5$, $b = -9$, and $c = 3$.

A. -23



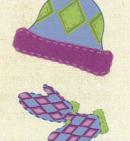
INTERNET

State Test Practice

Argyle Arithmetic

Each number in a purple diamond is the sum of the two numbers below it. Each number in a green diamond is the difference of the two numbers above it. Copy and complete the argyle arithmetic.





Problem Solving Strategies

Guess, Check, and Revise

Draw a Diagram

Act It Out

Work Backward

Look for a Pattern

Make a Table

Solve a Simpler Problem

Look for a Pattern

Problem Greg is setting up for a craft fair and is stacking gift boxes for a display. He knows a pyramid with a height of 2 boxes contains 5 boxes, a pyramid with a height of 3 boxes contains 14 boxes, and a pyramid with a height of 4 boxes contains 30 boxes. How many boxes will he need to make a pyramid with a height of 6 boxes?

Read and Understand

Read the problem carefully.

You need to find how many boxes Greg needs to make a pyramid with a height of 6 boxes.

Make a Plan

Decide on a strategy to use.

Sketch or model a pyramid with 5 boxes and a pyramid with 14 boxes. Look for a pattern. You can follow the pattern to determine how many boxes Greg will need.

Solve the Problem

Reread the problem and look for a pattern.

Sketch or model several rectangular pyramids and count how many boxes are at each level. Make a table that shows the number of boxes in each level.

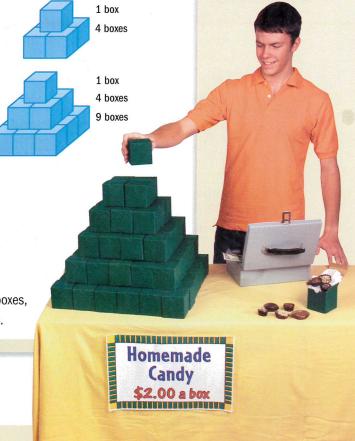
Level	Boxes in	Boxes in
n	level n	<i>n</i> -level pyramid
1	12	$1^2 = 1$
2	22	$1^2 + 2^2 = 5$
3	3 ²	$1^2 + 2^2 + 3^2 = 14$

The number of boxes in each level is the square of the number of the level. The number of boxes in a pyramid is the sum of the squares.

ANSWER To make a rectangular pyramid with a height of 6 boxes, Greg will need $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$ boxes.



Sketch a top view of each level to check your answer.



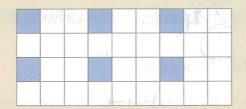
Practice the Strategy

Use the strategy look for a pattern.

1. Geometry Make a table showing the number of dots in each triangle. Determine the number of dots in the ninth triangle in this sequence.



- 2. Track Your track coach tells you to do a running drill in which you run 16 feet and run back. Then you run 32 feet and run back. Next you run 48 feet and run back. How far would you expect to run next?
- 3. Tiling Jason and Emily are laying tiles in the kitchen of a restaurant that measures 36 feet by 24 feet. Each tile measures 1 foot by 1 foot. If they follow the pattern shown below, how many blue tiles will they need to fill the entire kitchen with this pattern?



4. Number Sense Copy and complete the table below. Look for a pattern so you can evaluate 11 • 97 using mental math.

11 • 12	132	12 + 120
11 • 13	143	13 + ?
11 • 14	154	14 + <u>?</u>
11 • 15	165	15 + <u>?</u>
11 • 16	176	16 + <u>?</u>



Mixed Problem Solving

Use any strategy to solve the problem.

- **5. Who's Oldest?** Scott is two years older than Anne, and Kelly is three years younger than Scott. Ben is nine years less than twice Scott's age, and Anne is 10 years old. Determine the ages of Scott, Ben, and Kelly.
- 6. Baking You have a recipe that makes 24 cookies. The ingredients include 2 eggs, 1 cup of sugar, 1.5 cups of flour, 1 teaspoon of vanilla, and 1 teaspoon of baking soda. What is the greatest number of cookies you can make if you have 12 eggs, 4 cups of sugar, and 9 cups of flour?
- **7. Stock Prices** A newspaper reports these changes in the price of a stock during a 5-day period: -1, -8, +2, -4, and +6. The stock price ended at \$35 on the fifth day. How much was the price of a stock before the 5-day period started?
- 8. Cereal You are stacking boxes of cereal for a display. You use a total of 78 boxes, and each row has one fewer box than the row below it. How many rows make up the display if the top row has one box?
- 9. Basketball In a basketball game, there are 1 point free throws, 2 point field goals and 3 point field goals. How many ways can you score 12 points?



Multiplying Integers

BEFORE

Now

WHY?

You added and subtracted integers.

You'll multiply integers.

So you can find the worth of a coin in a game, as in Ex. 32.

Word Watch

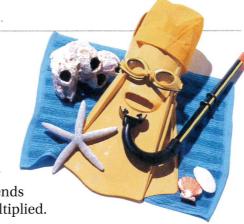
Review Words

integer, p 53 product, p. 713

In the Real World

Diving A diver is exploring a coral reef. The diver's depth is changing by -6 feet per second. If the diver started at sea level, what is the diver's position after 10 seconds?

To find the position, you can multiply integers. When you multiply integers, the sign of the product depends on the signs of the integers being multiplied.





Multiplying Integers

Words

The product of two integers with the same sign is positive.

The product of two integers with different signs is negative.

Numbers

$$4 \cdot 2 = 8$$

 $-3 \cdot (-7) = 21$

$$4 \cdot (-2) = -8$$

$$-3 \cdot 7 = -21$$



Multiplying Integers

To find the diver's position relative to sea level after 10 seconds, use the distance formula d=rt.

d = rt

Write the distance formula.

d = -6(10)

Substitute -6 for r and 10 for t.

d = -60

Different signs, so product is negative.

ANSWER The diver's position relative to sea level is -60 feet.

Your turn now

Use the information above.

1. Find the diver's position relative to sea level after 13 seconds.



Multiplication Properties

Multiplication Property of Zero

Words The product of an integer and 0 is 0.

Numbers $-4 \cdot 0 = 0$ **Algebra** For any value of a, $a \cdot 0 = 0$.

Identity Property of Multiplication

Words The product of an integer and 1 is the integer.

Numbers 4(1) = 4**Algebra** For any value of a, a(1) = a.

When you multiply a number by -1, the product is the *opposite* of the original number.

with Solving

When you multiply more than two positive or negative integers:

- If there is an even number of negative factors then the product is positive.
- If there is an odd number of negative factors then the product is negative.

EXAMPLE 2 Multiplying Two or More Integers

a.
$$-1(6) = -6$$

b. -8(-2) = 16

c. -15(0) = 0

d. 4(-10)(-12) = -40(-12)

= 480

Different signs, so product is negative.

Same sign, so product is positive.

Product of an integer and 0 is 0.

Multiply from left to right.

Multiply.

EXAMPLE 3 Evaluating an Expression with Integers

Evaluate $a^2 + 3b$ when a = -5 and b = -11.

$$a^2 + 3b = (-5)^2 + 3(-11)$$

Substitute -5 for a and -11 for b.

$$=$$
 25 $+$ 3(-11)

Evaluate the power.

$$=25+(-33)$$

Multiply.

$$= -8$$

Add.

Your turn now

Find the product.

2.
$$-1(4)$$

3. 7(0)

4.
$$-6(-11)$$

4.
$$-6(-11)$$
 5. $-1(-12)(-9)$

Evaluate the expression when a = 3, b = -4 and c = -8.

6.
$$ac - b$$

7.
$$ac + b$$

8.
$$a^2 + bc$$

9.
$$ab - c^2$$



Getting Ready to Practice

1. Vocabulary Copy and complete: The product of a positive integer and a negative integer is a ? integer.

Find the product.

2.
$$-4(-7)$$

3.
$$0(-9)$$

4.
$$-3(6)$$

5.
$$-1(-2)(-3)$$

6.
$$2(-4)(5)$$

7.
$$10(-9)(-3)$$

- 8. Banking You have \$500 in a savings account. Over a 2 month period, you make 9 withdrawals of \$30 each. What is your new balance?
- 9. Find the Error Describe and correct the error in the solution.

$$-8(-12) = -96$$

with Homework

Example Exercises

1 10-21

2 10-21

3 22-25



- · More Examples
- · eTutorial Plus

Practice and Problem Solving

Find the product.

10.
$$-6(7)$$

11.
$$-1(-17)$$

13.
$$-4(-11)$$

16.
$$-15(-12)$$

17.
$$-1(-32)$$

18.
$$-2(5)(-6)$$

20.
$$-8(-7)(-5)$$
 21. $12(0)(-45)$

Evaluate the expression when x = -9, y = -7, and z = -4.

22.
$$xy + z$$

23.
$$xy - y$$

25.
$$-3xy + 2yz$$

Find the product.

26.
$$|-2| \cdot 5$$

28.
$$-7(-8) \cdot |-4|$$

Mental Math Use mental math to solve the equation.

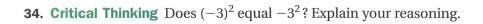
29.
$$2x = -8$$

30.
$$-21y = 63$$

31.
$$-5(-4)z = -80$$

- **32.** Video Game David is playing a video game. If he falls into a pit, he loses 125 points. If he collects coins, he will gain points. He has 400 points before he falls into 3 pits. After he collects a coin in each pit, his score is 175 points. How many points is each coin worth?
- **33.** Look for a Pattern Evaluate $(-10)^1$, $(-10)^2$, $(-10)^3$, $(-10)^4$, and $(-10)^5$. How is the exponent related to the sign of the power?







INTERNET

State Test Practice

Evaluate the expression when a = -8 and b = -11.

35.
$$-a(-a)$$

36.
$$a(-b^2)b$$

37.
$$[a + (-a)b]^2$$



38. Check for Reasonableness A coconut falls 100 feet from a palm tree. The equation $h = -16t^2 + 100$ gives the height h, in feet, of the coconut after falling for t seconds. Evaluate the equation when t equals 2, 2.5, and 3 seconds. When does the coconut hit the ground? What is the actual height of the coconut after 3 seconds?



39. Stock Market Your uncle owns 25 shares of stock A, 45 shares of stock B, and 60 shares of stock C. In one day, the price per share changed by +\$.56 for stock A, -\$1.46 for stock B, and -\$.50 for stock C. Find the total change in value of your uncle's stock.

Evaluate the expression when w = -31, y = 52, and z = -63.

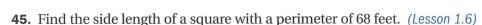
41.
$$yz - wyz$$

42.
$$3yz - wy$$

41.
$$yz - wyz$$
 42. $3yz - wy$ **43.** $-2wy - 2wz$

44. Challenge The product of a number and its *multiplicative inverse* is 1. For example, $4 \cdot \frac{1}{4} = 1$, so 4 and $\frac{1}{4}$ are multiplicative inverses. Give an example of a negative number and its multiplicative inverse.

Mixed Review



Find the difference. (Lesson 2.3)

Basic Skills Find the quotient.

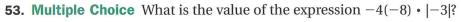
49.
$$75 \div 5$$

50.
$$0 \div 12$$

50.
$$0 \div 12$$
 51. $34 \div 17$ **52.** $63 \div 7$

Test-Taking Practice





- 54. Multiple Choice When you multiply an integer less than 1 and an integer less than -1, the product is which of the following?
 - **F.** less than zero

- **G.** greater than zero
- **H.** less than or equal to zero
- I. greater than or equal to zero



Dividing Integers

BEFORE

Now

WHY?

You added, subtracted, and multiplied integers.

You'll divide integers.

So you can convert temperatures, as in Ex. 31.

In the Real World



mean, p. 75

Temperatures One of the coldest places on Earth is the Russian town of Verkhoyansk, located near the Arctic Circle. The table shows the average high temperatures in Verkhoyansk. What is the average of these temperatures?

	Winter Temperatures				
	Month	Average High			
	December	−44°F			
10000	January	−48°F			
	February	−38°F			
	March	−7°F			

You will use the rules for dividing integers to find an average temperature in Example 2. These rules are similar to the rules for multiplying integers.



Dividing Integers

Words

The quotient of two integers with the same sign is positive.

The quotient of two integers with different signs is negative.

The quotient of zero and any nonzero integer is 0.

Numbers

$$\frac{12}{6} = 2$$
 $\frac{-12}{-6} = 2$

$$\frac{12}{-6} = -2 \qquad \frac{-12}{6} = -2$$

$$\frac{0}{12} = 0$$
 $\frac{0}{-12} = 0$

You cannot divide a number by 0. Any number divided by 0 is undefined.

vatch Our

EXAMPLE 1 Dividing Integers

a.
$$\frac{-40}{-8} = 5$$

Same sign, so quotient is positive.

b.
$$\frac{-14}{2} = -7$$

b. $\frac{-14}{2} = -7$ Different signs, so quotient is negative. **c.** $\frac{36}{-9} = -4$ Different signs, so quotient is negative.

c.
$$\frac{36}{-9} = -4$$



Temperatures

One of the lowest temperatures on Earth, -90°F, was recorded in Verkhoyansk, Russia, on February 7, 1892. How much colder is this than today's temperature where you live?

The **mean** of a data set is the sum of the values divided by the number of values.

$$mean = \frac{sum of values}{number of values}$$

EXAMPLE 2 Finding a Mean

Temperatures To find the mean of the monthly average high temperatures in Verkhoyansk, Russia, given on page 74, first find the sum of the temperatures.

$$-44 + (-48) + (-38) + (-7) = -137$$

Then, divide the sum by the number of temperatures.

$$\frac{-137}{4} = -34.25$$

ANSWER The mean of the temperatures is about -34° E.

Your turn now Find the quotient.

1.
$$\frac{-33}{11}$$

2.
$$\frac{-25}{-5}$$
 3. $\frac{0}{-4}$ **4.** $\frac{72}{-9}$

3.
$$\frac{0}{-4}$$

4.
$$\frac{72}{-9}$$

Find the mean of the data.

EXAMPLE 3 Evaluating Expressions

Evaluate the expression when a = -24, b = 8, and c = -4.

a.
$$\frac{a}{b}$$

b.
$$\frac{ab}{c}$$

Solution

a.
$$\frac{a}{b} = \frac{-24}{8}$$
 Substitute values.

$$= -3$$
 Different signs, so quotient is negative.

b.
$$\frac{ab}{c} = \frac{-24 \cdot 8}{-4}$$
 Substitute values.

$$= \frac{-192}{-4}$$
 Multiply.



Getting Ready to Practice

1. Vocabulary Copy and complete: To find the ? of three numbers, add them and divide the sum by three.

Find the quotient.

2.
$$\frac{-44}{4}$$

3.
$$\frac{0}{-7}$$

2.
$$\frac{-44}{4}$$
 3. $\frac{0}{-7}$ **4.** $\frac{-81}{-9}$ **5.** $\frac{50}{-10}$

5.
$$\frac{50}{-10}$$

Evaluate the expression when x = 18, y = -12, and z = -6.

6.
$$\frac{y}{z}$$

7.
$$\frac{x}{z}$$

8.
$$\frac{xz}{y}$$

9.
$$\frac{z^2}{y}$$

10. Guided Problem Solving Will opened a used musical instrument shop. The table shows his profits for the first three months. Find his mean profit for these months.

Month	Profit
October	-\$172
November	-\$203
December	\$157

- (1 Add the profits.
- (2 Count the number of months given.
- (3 Divide the sum in Step 1 by the number in Step 2. Should you give an exact or approximate answer?



Practice and Problem Solving

Find the quotient.

11.
$$\frac{-42}{-6}$$

12.
$$\frac{-28}{2}$$

13.
$$\frac{36}{-4}$$

11.
$$\frac{-42}{-6}$$
 12. $\frac{-28}{2}$ **13.** $\frac{36}{-4}$ **14.** $\frac{-19}{-1}$

15.
$$\frac{-49}{-7}$$

15.
$$\frac{-49}{-7}$$
 16. $\frac{-66}{-11}$ **17.** $\frac{0}{-18}$ **18.** $\frac{-27}{0}$

17.
$$\frac{0}{-18}$$

18.
$$\frac{-27}{0}$$

Find the mean of the data.

21. Writing Is the mean of a set of negative numbers *always*, *sometimes*, or never negative? Give an example to support your reasoning.

Evaluate the expression when m = 16, n = -8, and p = -32.

22.
$$\frac{m}{n}$$

23.
$$\frac{p}{n}$$

24.
$$\frac{n^2}{m}$$

23.
$$\frac{p}{n}$$
 24. $\frac{n^2}{m}$ **25.** $\frac{p}{n+m}$



26. Track and Field In five trial runs of a 100 meter dash, a runner has times of 14.01, 15.27, 16.17, 14.42, and 15.01 seconds. The table shows the time differences between the team average, in seconds, and the runner's times. Find the mean of the differences between the team average and the runner's times.

Trial	Difference	
1	-1.51	
2	-0.25	
3	0.65	
4	-1.1	
5	-0.51	
FIN	ISH	

Find the quotient.

27.
$$\frac{-9}{6}$$

28.
$$\frac{15}{-12}$$

29.
$$\frac{-8}{-10}$$

27.
$$\frac{-9}{6}$$
 28. $\frac{15}{-12}$ **29.** $\frac{-8}{-10}$ **30.** $\frac{-6}{-30}$

31. Reindeer The natural habitat of a reindeer is the Arctic tundra. The average temperature during the winter in the Arctic tundra is -34°C. You can convert degrees Celsius C to degrees Fahrenheit F by using the formula $F = \frac{9}{5}C + 32$. What is the average winter Arctic tundra temperature in degrees Fahrenheit?

Challenge Evaluate the expression when a = -15, b = 50, and c = 20.

32.
$$\frac{b^2+c}{c^2a}$$

33.
$$\frac{-2(a^2+c^2)}{b}$$
 34. $\frac{(b+c)^2}{a}$

34.
$$\frac{(b+c)^2}{a}$$

Mixed Review 🗘



Write the product as a power. (Lesson 1.4)

Find the product. (Lesson 2.4)

38.
$$-11(-8)$$

40.
$$6(-8)(-2)$$

Test-Taking Practice



- **41.** Multiple Choice When you multiply the quotient of a negative integer and a positive integer by -1, what is the sign of the product?
 - **A.** negative
- **B.** positive
- **C.** zero
- **D.** cannot be determined
- **42. Short Response** The table shows the temperatures in Fairbanks, Alaska. Calculate the mean temperature. Show your work or explain in words how you determined the mean. Should you give an exact or approximate answer?

Day	Temperature
Monday	−8°F
Tuesday	−1°F
Wednesday	−6°F
Thursday	−21°F
Friday	−31°F
Saturday	−34°F



Notebook Review



Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

integer, p. 53 negative integer, p. 53 positive integer, p. 53 absolute value, p. 54

opposite, p. 54 mean, p. 75

Use Your Vocabulary

1. Draw a number line. Label three positive integers and three negative integers. Graph the two integers that have an absolute value of 3.

2.1 Can you graph and order integers?



EXAMPLE Order the integers from least to greatest:

$$24, -6, -45, 0, 12, -20, -32.$$

In order from least to greatest: -45, -32, -20, -6, 0, 12, 24



✓ Use a number line to order the integers from least to greatest.

2.2-2.3 Can you add and subtract integers?



EXAMPLE Find the sum or difference.

a.
$$-22 + (-18) = -40$$

Same sign, so sum has common sign.

b.
$$19 - 34 = 19 + (-34)$$

Add the opposite of 34.

$$= -15$$

Add.



4.
$$-26 + 64$$

5.
$$37 + (-92)$$

6.
$$-41 + (-78)$$

8.
$$-51 - 14$$

9.
$$-44 - (-29)$$

2.4-2.5 Can you multiply and divide integers?



EXAMPLE Find the product or quotient.

a.
$$-7 \cdot 6 = -42$$

a. $-7 \cdot 6 = -42$ **Different signs, so product is negative.**

b.
$$\frac{-56}{-7} = 8$$

Same sign, so quotient is positive.

✓ Find the product or quotient.

12.
$$\frac{-90}{15}$$

11.
$$31(-2)$$
 12. $\frac{-90}{15}$ **13.** $\frac{-52}{-13}$

Stop and Think

about Lessons 2.1-2.5

14. Critical Thinking Is the opposite of the sum of two numbers equal to the sum of the opposites of the numbers? Explain.

Review Quiz 1

Copy and complete the statement with < or >.

Evaluate the expression.

5.
$$-6+1$$

6.
$$-20 + (-10)$$
 7. $-4 - (-3)$

9.
$$-12(4)$$
 10. $\frac{-48}{-8}$

10.
$$\frac{-48}{-8}$$

Find the mean of the data.

11.
$$-9$$
, -16 , 16 , 4 , 2 , -10 , 8 , 20
12. -10 , 6 , -11 , -6 , -7 , 3 , -4 , 1 , 1
13. 8 , -9 , -13 , 5 , -4 , -3 , -5
14. 5 , -6 , -10 , -15 , 7 , 9 , -1 , -3 , 4

Evaluate the expression when a = -2, b = 10, and c = -3.

15.
$$b - a$$

16.
$$a - c - b$$
 17. abc

18.
$$\frac{b}{a}$$

19. Gravity You drop a ball out of a window that is 144 feet above the ground. The equation $h = -16t^2 + 144$ gives the height h, in feet, of the ball after falling for t seconds. Find the height of the ball after 1, 2, 3, and 4 seconds. When does the ball hit the ground?



Number Properties

BEFOR

Now

WHY?

You evaluated expressions.

You'll use properties to evaluate expressions.

So you can find your weekly pay, as in Ex. 39.

In the Real World



Review Words

sum, p. 709 product, p. 713 **Tour Biking** You are going on a 400 mile bike trip. You plan to cycle at an average speed of 12 miles per hour for 7 hours a day. Can you complete the trip in 5 days?

The commutative properties of addition and multiplication can be used to make evaluating expressions using mental math easier.



The Commutative Property

Addition

Multiplication

Words

You can add numbers of a sum in any order.

You can multiply factors of a product in any order.

Numbers

$$3 + (-8) = -8 + 3$$

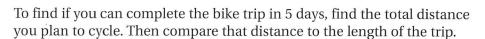
$$5(-6) = -6(5)$$

$$a + b = b + a$$

$$ab = ba$$



Using the Commutative Property





Total distance	= Average speed	Hours Number of days
	= 12 · 7 · 5	Substitute known values.
	$= 12 \cdot 5 \cdot 7$	Commutative property of multiplication
	= 60 • 7	Multiply.
	= 420	Multiply.
		miles hours

The unit for the result is miles. $\frac{\text{miles}}{\text{hour}} \cdot \frac{\text{hours}}{\text{day}} \cdot \text{days} = \text{miles}$

ANSWER Because 400 miles is less than the 420 miles you can travel in 5 days, you can complete the trip in 5 days.



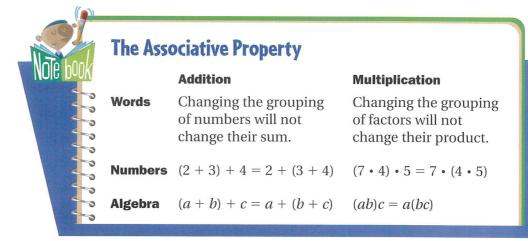
When deciding what numbers to add or multiply first, look for pairs whose sum or product ends in zero, because multiples of 10 are easier to work with. Subtracting a number is the same as adding the opposite, so you can write expressions to use the commutative property of addition.

EXAMPLE 2 Using the Commutative Property

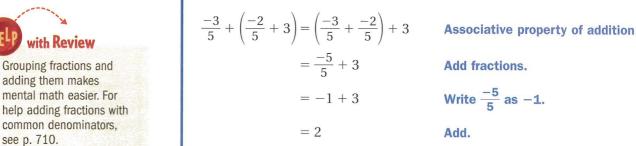
$$-54 + 35 - 16 = -54 + 35 + (-16)$$
 Change subtraction to addition
$$= -54 + (-16) + 35$$
 Commutative property of addition
$$= -70 + 35$$
 Add -54 and -16 .
$$= -35$$
 Add -70 and -35 .

Your turn now Use the commutative property to evaluate.

The associative properties of addition and multiplication can also be used to make evaluating expressions using mental math easier.



EXAMPLE 3 Using the Associative Property





Commute means change locations. You can use the commutative properties to change the order of numbers.

Associate means group together. You can use the associative properties to group numbers differently.

EXAMPLE 4 Using the Associative Property

$$5 \cdot (11 \cdot 2) = 5 \cdot (2 \cdot 11)$$

= $(5 \cdot 2) \cdot 11$
= $10 \cdot 11$

= 110

Commutative property of multiplication Associative property of multiplication

Multiply inside grouping symbols.

Multiply.

Your turn now Evaluate the expression using mental math.

5.
$$46 + (-63 - 46)$$

6.
$$-2(46 \cdot 50)$$

7.
$$4\left(\frac{1}{4} \cdot 23\right)$$

8.
$$\frac{3}{7} + \left(8 + \frac{4}{7}\right)$$

8.
$$\frac{3}{7} + \left(8 + \frac{4}{7}\right)$$
 9. $[-21 \cdot (-29)] \cdot 0$

Exercises More Practice, p. 728



Getting Ready to Practice

Vocabulary Match the equation with the property it illustrates.

1.
$$(x + 9) + 1 = x + (9 + 1)$$
 A. Identity property of addition

3.
$$8a = a \cdot 8$$

4.
$$-16 + 0 = -16$$

5.
$$(5 \cdot 7)y = 5(7y)$$

F. Identity property of multiplication

6.
$$-24 + a = a + (-24)$$

8.
$$43 + (-27) - 13$$
 \bigcirc **9.** $5 \cdot (-29) \cdot 2$

10.
$$-2(-9 \cdot 50)$$

11.
$$-53 + (-27 + 44)$$
 12. $[-4 \cdot (-7)](-5)$

12.
$$[-4 \cdot (-7)](-5)$$

13. Juice Box You have a juice box that is 2.5 inches long, 1.5 inches wide, and 4 inches high. The formula for the *volume* of a box is V = lwh. How much juice is in the box, in cubic inches?



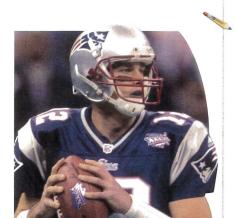


Example Exercises

1	14-29, 32



- · More Examples
- · eTutorial Plus



Practice and Problem Solving

Use the properties of addition and multiplication to find the missing number. Name the property.

14.
$$28 + ? = 65 + 28$$

15.
$$54 \cdot 16 = 16 \cdot ?$$

16.
$$(7 \cdot 3)3 = ? (3 \cdot 3)$$

17.
$$4 + (? + 2) = (4 + 9) + 2$$

Evaluate the expression. Justify each step.

18.
$$-86 + 29 + (-34)$$

19.
$$45 - (-68) - 44$$

18.
$$-86 + 29 + (-34)$$
 19. $45 - (-68) - 44$ **20.** $-57 - 38 - (-57)$

21.
$$12 + (-39 + 48)$$

21.
$$12 + (-39 + 48)$$
 22. $(-26 + 33) + (-4)$ **23.** $[25 \cdot (-7)]4$

24.
$$-40\left(\frac{1}{2} \cdot 45\right)$$

Algebra Simplify the expression.

27.
$$7 \cdot x \cdot 10$$

28.
$$-67 + [x + (-13)]$$
 29. $(52 + x) + 18$

29.
$$(52 + x) + 18$$

- **30. Writing** You need to find the sum of 52, 99, 65, 38, and 11. Explain how the commutative and associative properties of addition can help you find the sum using mental math.
- **31. Critical Thinking** Is division commutative? Justify your answer with an example.
- **32. Super Bowl** During Super Bowl XXXVI, six New England Patriots rushed the football for 92 yards, 22 yards, 15 yards, 5 yards, 3 yards, and -4 yards. What was the total number of their rushing yards?

Evaluate the expression. Show each step.

33.
$$3.6 + 5.7 + (-3.6)$$

34.
$$\frac{1}{2} \cdot 17 \cdot 2$$

33.
$$3.6 + 5.7 + (-3.6)$$
 34. $\frac{1}{2} \cdot 17 \cdot 20$ **35.** $5(7 \cdot 4)(0.25)$ **36.** $\left(-\frac{2}{7} + 5\right) + \frac{3}{7}$ **37.** $12 \cdot (7 \cdot 1 \cdot 5)$ **38.** $\left(\frac{2}{3} \cdot 7\right) \cdot 21$

38.
$$\left(\frac{2}{3} \cdot 7\right) \cdot 2$$

39. Paycheck The table shows the hours you worked during one week. Your hourly wage is \$6 per hour. Use the commutative property of multiplication to find the amount you earned for the week.

Time Card			
Day	Time in	Time out	
Monday	4 P.M.	6 P.M.	
Tuesday	4 P.M.	6 P.M.	
Wednesday	_	-	
Thursday	3 P.M.	5 P.M.	
Friday	3 P.M.	5 P.M.	
Saturday	11 A.M.	1 P.M.	

40. Compare Is the expression -15 + 34 - 44 - 19 + 51 equivalent to the expression 34 - 19 + 15 - 44 + 51? Explain your reasoning.



41. Critical Thinking Explain how the student used the commutative property of addition to go from the first expression to the second expression. Use the same method to find the sum of the numbers from 1 to 19.

$$1+2+3+4+5+6+7+8+9=10+10+10+10+5$$

= 45

42. Sale Price You are buying 8 yards of fabric that costs \$5.25 per yard. You have a coupon for half off the original price. What is the price of your purchase after the discount?



43. Challenge Use the commutative properties of addition and multiplication to write three expressions equivalent to $4 \cdot 8 + 5$.

Mixed Review 📢

44. Temperature During the first 10 days of January, 2002, the daily low temperatures in Alaska, in degrees Fahrenheit, were -29° , -24° , -18° , -42° , -42° , -42° , -40° , -44° , -26° and -19° . Find the mean daily low temperature for these 10 days. (Lesson 2.5)

Choose a Strategy Use a strategy from the list to solve the following problem. Explain your choice of strategy.

45. You have two stacking bookcases that are 60 inches tall when stacked on top of each other. If you place them side by side, the difference of their heights is 8 inches. How tall is each bookcase?

Problem Solving Strategies

- Guess, Check, and Revise
- Look for a Pattern
- Work Backward

Basic Skills Use a number line to compare the numbers.

- **46.** 1.2 and 0.8
- **47.** 1.35 and 1.53
- **48.** 0.24 and 0.25

Test-Taking Practice 🐚



49. Multiple Choice Which of the following is equivalent to 7-5-(-4)+6?

A.
$$7-6-(-4)+5$$

B.
$$5-7-(-4)+6$$

C.
$$7 - 5 + 2$$

D.
$$7 + (-5) + 4 + 6$$

50. Short Response You need to find the product of 25, 6, 4, and 7. Explain how the commutative and associative properties of multiplication can help you find the product using mental math.

State Test Practice



The Distributive Property

BEFORE

Now

WHY?

You used addition and multiplication properties.

You'll use the distributive property.

So you can find the cost of souvenirs, as in Ex. 25.

In the Real World



distributive property, p. 85 terms, p. 86 like terms, p. 86 coefficient, p. 86 constant term, p. 86 Architecture A replica of the Parthenon, an ancient temple in Greece, was built in Nashville, Tennessee, in 1897. The diagram below shows the approximate dimensions of two adjacent rooms inside the replica. How can you find the total area of the two rooms?



EXAMPLE 1

Finding a Combined Area

Two methods can be used to find the total area of the two rooms.

Method 1 Find the area of each room, then find the total area.

Area =
$$63(44) + 63(93)$$

$$= 2772 + 5859$$

= 8631 square feet

Method 2 Find the total length, then multiply by the common width.

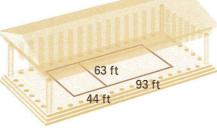
Area =
$$63(44 + 93)$$

$$= 63(137)$$

= 8631 square feet

ANSWER The total area of the two rooms is 8631 square feet.

Example 1 demonstrates the distributive property.





The Distributive Property

Algebra
$$a(b+c) = ab + ac$$
 Numbers $6(4+3) = 6(4) + 6(3)$

$$a(b-c) = ab - ac$$

$$7(8-5) = 7(8) - 7(5)$$

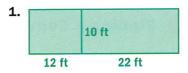
The distributive property can be applied to expressions involving a sum or difference of two or more numbers or variable expressions.

EXAMPLE 2 Using the Distributive Property

a.
$$-5(x + 10) = -5x + (-5)(10)$$
 Distributive property $= -5x + (-50)$ Multiply.

b.
$$3[1-20+(-5)] = 3(1) - 3(20) + 3(-5)$$
 Distributive property
$$= 3 - 60 + (-15)$$
 Multiply.
$$= 3 + (-60) + (-15)$$
 Add the opposite of 60.
$$= -72$$
 Add.

Your turn now Write two expressions for the total area of the two rectangles. Find the total area.





Use the distributive property to evaluate or simplify the expression.

3.
$$-2(5+12)$$

3.
$$-2(5+12)$$
 4. $-4(-7-10)$ **5.** $2(w-8)$

5.
$$2(w-8)$$

6.
$$-8(z+25)$$

with Notetaking

When you add new vocabulary words to your notebook be sure to include examples of how they are used.

Combining Like Terms In a sum the parts that are added together are the terms of the expression. You can use the distributive property to combine *like terms*. Like terms have identical variable parts raised to the same power. In a term the number multiplied by the variable is the **coefficient** of the variable. A term that has no variable is a constant term .

Coefficients are 4 and 8. Constant term is 1.
$$4x + 8x + 1$$

$$4x \text{ and } 8x \text{ are like terms.}$$

EXAMPLE 3 Combining Like Terms

a.
$$3x + 4x = (3 + 4)x$$
 Distributive property $= 7x$ Add inside grouping symbols.

b.
$$-9y + 7y + 5z = (-9 + 7)y + 5z$$
 Distributive property
$$= -2y + 5z$$
 Add inside grouping symbols.

You may need to use the distributive property before you can combine like terms.



Remember that $x = 1 \cdot x$, so x has a coefficient of 1.

EXAMPLE 4 Simplifying an Expression

a.
$$2(4 + x) + x = 8 + 2x + x$$
 Distributive property

$$= 8 + 3x$$
 Combine like terms.

b.
$$-5(3x-6) + 7x = -15x + 30 + 7x$$
 Distributive property $= -8x + 30$ Combine like terms.

Your turn now Simplify the expression by combining like terms.

7.
$$4x - 7x$$

8.
$$5v + 9z - 7 - 3t$$

8.
$$5y + 9z - 7 - 3y$$
 9. $5(x - 6) + 3x + 4$

Exercises More Practice, p. 728



Getting Ready to Practice

1. Vocabulary Identify any *like terms* and *coefficients* in the expression 7x - 3y - 6y + x + 2.

Use the distributive property to write an equivalent expression.

2.
$$4(7 + 8)$$

3.
$$-7(3+2)$$

Simplify the expression by combining like terms.

5.
$$3y + 6y$$

6.
$$9a - 4b + a$$

7.
$$8m + n - 2m - 4b$$

8. Guided Problem Solving You are buying three pairs of flip-flops that cost \$12.90 each. Use mental math and the distributive property to find the total cost of the flip-flops.



- (2 Find the products 3(13) and 3(0.10).
- (3 Find the difference of the products.





Example Exercises

- 1 33,44
- 2 9-18, 27-32
- 3 19-24, 27-32
- 4 19-24, 27-32



- · More Examples
- · eTutorial Plus





Practice and Problem Solving

Match the expression with its simplified expression.

9.
$$3(x+4)$$

10.
$$4(x+3)$$

11.
$$2(2x-6)$$

12.
$$x(4+9)$$

A.
$$4x - 12$$

B.
$$3x + 12$$

C.
$$13x$$

D.
$$4x + 12$$

Use the distributive property to evaluate or simplify the expression.

13.
$$9(x-3)$$

14.
$$-12(4+5+y)$$
 15. $8(5+2)$

15.
$$8(5+2)$$

16.
$$19[7 + w + (-2)]$$
 17. $-34(z - 21 - 5)$ **18.** $-13(-12 + 9)$

17.
$$-34(z-21-5)$$

18.
$$-13(-12+9)$$

Simplify the expression by combining like terms.

19.
$$r + 2s + 3r$$

20.
$$11w + 9z + 3z + 5w$$

21.
$$7a - 2a + 8b - 2b$$

22.
$$3x + 2x + y + 2y - 3$$

23.
$$-3x + 2x - 9y - 2x$$

24.
$$r + 2s - (-3r) - s$$

25. Souvenirs You are on vacation in Massachusetts and want to buy souvenirs for 6 friends at home. You decide to buy a trading pin that costs \$2.35 and a pen that costs \$.65 for each friend. Explain how to use mental math to find how much money you will need.



26. Find the Error Describe and correct the error in the solution.

$$4x + 2y - 7y - 2x = 4x - 2x + 2y - 7y$$

$$= 2x - 5y$$

$$= -3xy$$

Simplify the expression.

27.
$$7(y-2)$$

28.
$$9x(4-2+6)$$

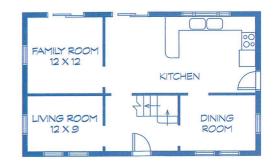
28.
$$9x(4-2+6)$$
 29. $3x+2-5x+4x$

30.
$$5(z+2z)-4z$$

30.
$$5(z+2z)-4z$$
 31. $8d-2(3d-5d)$

32.
$$4(3c-4)+2-4c$$

33. Area A floor plan of a house is shown. You want to carpet the family room and the living room. The carpeting you want to use in these two rooms sells for \$3.12 per square foot. How much will the carpet cost?



Simplify the expression by combining like terms.

34.
$$3a + 4b - 5 - a + 7b + 3 - b$$
 35. $5(x + 2) - 5(y + 3) - 2x + 5y$

35.
$$5(x+2) - 5(y+3) - 2x + 5y$$

36.
$$3.2(2z-3x)+4(1.1y+x)-2z$$
 37. $7(y-1.3)+2.4-5.3y$

37.
$$7(y-1.3) + 2.4 - 5.3y$$

38. Writing Explain what you should consider when deciding which expression is easier to evaluate, 8(1000 - 2) or $8 \cdot 1000 - 8 \cdot 2$. Give an example of a problem you think is easier to solve using the distributive property and an example you think is not easier to solve using the distributive property.

Mental Math Use mental math to find the product using a method like the one in Exercise 8 on page 87. Explain your reasoning.



40. 9(19)

41. 24(12)

42. 65(24)

- **43. Critical Thinking** Are 3xy and 4yx like terms? Explain your reasoning.
- **44.** Mural Some students have been given permission to paint murals on 5 walls at your school. The walls are all 8 feet tall. The mural widths are 21.5 feet, 35 feet, 27.5 feet, 33.5 feet, and 22.5 feet. Write two expressions to find the total area of the murals. Then find the total area.
- **45. Challenge** You are ordering T-shirts with your school logo. Each T-shirt costs \$7.25. There is a \$25 setup fee for silk-screening, and a screening charge of \$1.85 per shirt. Write an expression to find the total cost for x T-shirts. What is the total cost for 75 T-shirts? for 170 T-shirts?

Mixed Review 📢



Order the integers from least to greatest. (Lesson 2.1)

Evaluate the expression. (Lesson 2.6)

48.
$$-5(4 \cdot 17)$$

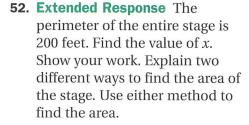
49.
$$(-23 + 14) - 12$$

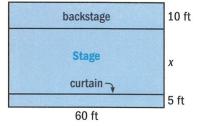
50.
$$17 + (3 - 12) + 24(0)$$

51.
$$(-4 \cdot 7) \cdot 25$$

Test-Taking Practice







Lesson 2.7 The Distributive Property



INTERNET

CLASSZONE.COM

State Test Practice

Natch Our

performs subtraction. Use the button to enter a

negative number.

Technology Activity

Using Integer Operations

GOAL Use a calculator to evaluate expressions.

Example

Use a calculator to evaluate the expression.

a.
$$-900,018 + (-805,560)$$
 b. $\frac{-278 \cdot (-640)}{-139}$

b.
$$\frac{-278 \cdot (-640)}{-139}$$

Solution

Use the following keystrokes to find your answer.

Keystrokes

Display

-1705578

ANSWER
$$-900,018 + (-805,560) = -1,705,578$$

Keystrokes

Display





ANSWER
$$\frac{-278 \cdot (-640)}{-139} = -1280$$

Your turn now Use a calculator to evaluate the expression.

1.
$$18,432 + (-46,978)$$
 2. $-50,215 + 1315$

7.
$$3336(-198 \cdot 398)$$
 8. $\frac{-105,638}{-221}$

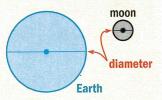
complete one orbit?

$$\frac{-105,638}{-221}$$

9. $\frac{-67,771}{671}$

11. Earth The diameter of Earth is about 4 times the diameter of the moon. The diameter of the moon is 3476 kilometers. What is the diameter of Earth? Find the difference of the diameter of Earth and

the diameter of the moon.





The Coordinate Plane

BEFORE

Now

WHY?

You used number lines.

You'll identify and plot points in a coordinate plane.

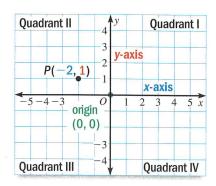
So you can predict the price of a phone call, as in Ex. 29.



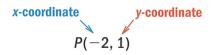
coordinate plane, p. 91 x-axis, y-axis, p. 91 origin, p. 91 quadrant, p. 91 ordered pair, p. 91 x-coordinate, p. 91 y-coordinate, p. 91

A **coordinate plane** is formed by the intersection of a horizontal number line called the **x-axis** and a vertical number line called the **y-axis**.

The axes meet at a point called the **origin** and divide the coordinate plane into four **quadrants**.



Points in a coordinate plane are represented by **ordered pairs**. The first number is the x-coordinate. The second number is the y-coordinate. Point P above is represented by the ordered pair (-2, 1).



EXAMPLE 1

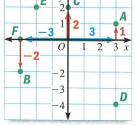
Naming Points in a Coordinate Plane

Give the coordinates of the point.

- **a.** *A*
- **b.** *B*
- **c.** C

Solution

a. Point *A* is 3 units to the right of the origin and 1 unit up. So, the *x*-coordinate is 3 and the *y*-coordinate is 1. The coordinates of *A* are (3, 1).



- **b.** Point *B* is 3 units to the left of the origin and 2 units down. So, the *x*-coordinate is -3 and the *y*-coordinate is -2. The coordinates of *B* are (-3, -2).
- **c.** Point C is 2 units up from the origin. So, the x-coordinate is 0 and the y-coordinate is 2. The coordinates of C are (0, 2).

Your turn now

Give the coordinates of the point.

1. D

2. E

3. *F*

EXAMPLE 2 Graphing Points in a Coordinate Plane

Plot the point and describe its location.

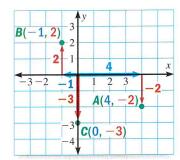
a.
$$A(4, -2)$$

b.
$$B(-1, 2)$$

c.
$$C(0, -3)$$

Solution

- **a.** Begin at the origin, move 4 units to the right, then 2 units down. Point A lies in Quadrant IV.
- **b.** Begin at the origin, move 1 unit to the left, then 2 units up. Point B lies in Quadrant II.
- **c.** Begin at the origin, move 3 units down. Point Clies on the y-axis.



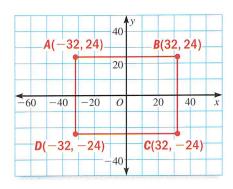
EXAMPLE 3 Finding Perimeter

Identify the figure and find its perimeter.

Solution

Points A, B, C, and D form a rectangle.

To find the length *l*, find the *horizontal* distance from *A* to *B*.



$$l = |x$$
-coordinate of $A - x$ -coordinate of $B|$
= $|-32 - 32| = |-64| = 64$

To find the width w, find the vertical distance from A to D.

$$w = |y$$
-coordinate of $A - y$ -coordinate of $D|$
= $|24 - (-24)| = |48| = 48$

Perimeter =
$$2l + 2w = 2(64) + 2(48) = 224$$

ANSWER The rectangle has a perimeter of 224 units.



You need to use absolute value signs because length and width are always positive.

with Solving

Points on the x-axis or

y-axis do not lie in any

quadrant.

Your turn now Plot the point and describe its location.

4.
$$R(-3, 4)$$

6.
$$T(0,3)$$

7.
$$U(-4,0)$$

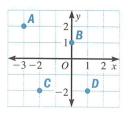
8. Plot and connect points A(-20, 25), B(25, 25), C(25, -20), and D(-20,-20). Identify the resulting figure and find its perimeter.



Getting Ready to Practice

1. Vocabulary Draw a coordinate plane and label the *x*-axis, *y*-axis, each quadrant, and the origin.

Give the coordinates of the point.



Plot the point in a coordinate plane.

9.
$$(-2, -1)$$

10. City Park The rectangle with corners *A*, *B*, *C*, and *D* represents a city park. Find the distance around the city park if the length and width of each small square on the coordinate grid represents 100 feet.





Example Exercises

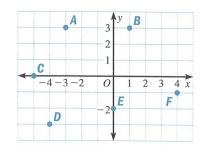
1	11-16



- · More Examples
- · eTutorial Plus

Practice and Problem Solving

Give the coordinates of the point.

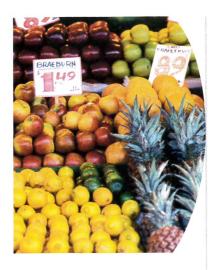


Plot the point in a coordinate plane and describe its location.

Plot and connect the given points. Then identify the resulting figure and find its perimeter.

21.
$$A(-2, 6)$$
, $B(2, 6)$, $C(2, -6)$, $D(-2, -6)$

22.
$$J(5, 4), K(5, -2), L(-1, -2), M(-1, 4)$$



23. Fruit The following ordered pairs represent the cost of buying several weights of pineapple from a fruit stand. The *x*-coordinate represents the number of pounds and the *y*-coordinate represents the cost.



(1, \$3.50), (2, \$7.00), (3, \$10.50), (4, \$14.00)

Plot the points in a coordinate plane. Identify the pattern. Use the pattern to estimate the cost of $2\frac{1}{2}$ pounds of pineapple.

Plot and connect the points. Find the perimeter and area of the rectangle formed.

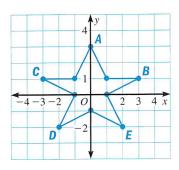
- **24.** Q(5, 2), R(5, -5), S(-3, -5), T(-3, 2)
- **25.** K(0, -5), L(-5, -5), M(-5, 0), N(0, 0)
- **26.** W(-1, -7), X(-1, 3), Y(2, 3), Z(2, -7)

Extended Problem Solving In Exercises 27–29, suppose your phone card charges 7 cents per minute for a phone call.

- **27. Calculate** Find the cost of a 10 minute call, a 20 minute call, and a 30 minute call.
- **28. Graph** Plot the costs you found in Exercise 27 in a coordinate plane, where the *x*-coordinate represents the length (in minutes) of the call and the *y*-coordinate represents the total cost of the call, in dollars.



- **29. Estimate** Draw a line through the points. Use the line to estimate the cost of an hour-long call.
- **30. Challenge** Draw a rectangle that has points in all four quadrants. Multiply each coordinate by 2 and draw this rectangle. Compare this rectangle with the original one and compare their perimeters. What do you find?
- **31. Slide** Copy the figure. Move the figure 2 units to the left and 3 units up. Give the new coordinates of *A*, *B*, *C*, *D*, and *E*.



Mixed Review 😂

Find the sum. (Lesson 2.2)

32.
$$24 + (-9) + (-12)$$
 33. $-14 + 30 + (-17)$ **34.** $-40 + 8 + 12$

33.
$$-14 + 30 + (-17)$$

34.
$$-40 + 8 + 12$$

Simplify the expression. (Lesson 2.7)

35.
$$4(x + 9)$$

36.
$$6y(8-6)$$

37.
$$-9(z-2)$$

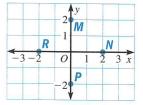
38. Basic Skills You are selling T-shirts as a fundraiser for your school's soccer team. You need to sell 35 shirts to reach your goal, and you have sold 18 shirts. How many more T-shirts do you need to sell?

Test-Taking Practice



- **39. Short Response** On a coordinate plane, plot the points A(2, 3), B(2, 7), C(6, 7), and D(6, 3). Connect the points to form a square. Imagine that this square moves 4 units up. Write the new coordinates for points A, B, C, and D.
- 40. Multiple Choice Which labeled point shown has an x-coordinate of 2?

$$\mathbf{D}.$$
 R





INTERNET

State Test Practice

Spatial Delivery

Jack delivers balloons in a city. Below is a grid of Jack's city and a list of the deliveries he made today. Copy the diagram on graph paper and plot each stop. How many blocks did Jack travel?

Delivery Stops

Began at Main and State Street

- 1. E 3rd St. and Main
- 2. E 3rd St. and N 2nd Ave.
- 3. E 1st St. and N 2nd Ave.
- 4. E 1st St. and S 2nd Ave.
- 5. W 2nd St. and S 3rd Ave.
- 6. W 3rd St. and S 1st Ave.
- 7. W 3rd St. and N 2nd Ave.
- 8. W 1st St. and N 2nd Ave.

+	+	+	eet	-N3-		-	_	+
+	+	+	State Street	-N2-		-	_	+
		+	Stat	-N1-				+
Main Street W3 W2 W1			Main Street					
			Ĺ		1	E2	2	E3
			State Street	-S1-				
+	+		te S	-S2-		_	-	+
			ta	-S3-				





Notebook Review



Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

distributive property, p. 85

terms, like terms, p. 86 coefficient, p. 86

constant term, p. 86 coordinate plane, p. 91 x-axis, y-axis, p. 91 origin, p. 91

quadrant, p. 91 ordered pair, p. 91 x-coordinate, p. 91 y-coordinate, p. 91

Use Your Vocabulary

1. What is the *y*-coordinate of the point (5, -5)?

2.6 Can you use commutative and associative properties?



EXAMPLE Evaluate the expression.

$$(-3+5)-7 = (-3+5)+(-7)$$

$$= [5+(-3)]+(-7)$$

$$= 5+[-3+(-7)]$$

$$= 5+(-10)$$

$$= -5$$

Change subtraction to addition. Commutative property of addition Associative property of addition Add inside grouping symbols. Add.



2.
$$-5(19 \cdot 2)$$

4.
$$-42 + (-18 - 23)$$

2.7 Can you use the distributive property?



EXAMPLE Simplify the expression.

$$6(2x + 7) - 9x = 12x + 42 - 9x$$
$$= 12x + 42 + (-9x)$$
$$= 12x + (-9x) + 42$$
$$= 3x + 42$$

Distributive property

Change subtraction to addition.

Commutative property of addition

Combine like terms.



5.
$$9(3a + 11) - 4$$

5.
$$9(3a + 11) - 4$$
 6. $-8b + 12(7b + 3)$ **7.** $2c - 8(9c - 5)$

7.
$$2c - 8(9c - 5)$$

2.8 Can you identify and plot points?

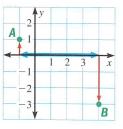


EXAMPLE Plot the point and describe its location.

- **a.** A(-1, 1)
- **b.** B(4, -3)

Solution

- **a.** Begin at the origin, move 1 unit to the left and 1 unit up. Point A lies in Quadrant II.
- **b.** Begin at the origin, move 4 units to the right and 3 units down. Point B lies in Quadrant IV.





3 3 Plot the point and describe its location.

- **8.** L(3, -5)
- **9.** M(0, -1)
- **10.** N(6, 0)
- **11.** P(-4, 2)

Stop and Think

about Lessons 2.6-2.8

🔪 **12. Writing** Explain how you can use the distributive property and mental math to evaluate the expression 4 • 6.11.

Review Quiz 2

Evaluate the expression. Justify each step.

Simplify the expression.

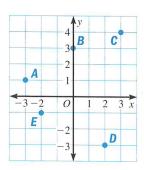
4.
$$9x + 22x$$

5.
$$3a - 2b + 6 - a$$

6.
$$8(y+2)-4y$$

In Exercises 7-9, use the coordinate plane.

- **7.** Write the coordinates of points *A*, *B*, *C*, *D*, and *E*.
- **8.** Which point lies on the γ -axis?
- **9.** Which point lies in Quadrant II?



10. Basketball In a basketball game Joe scored 13 points in the first quarter, 6 points in the second quarter, 7 points in the third quarter, and 14 points in the fourth quarter. How many points did Joe score?

Chapter Review



integer, p. 53 negative integer, p. 53 positive integer, p. 53 absolute value, p. 54 opposite, p. 54 mean, p. 75 distributive property, p. 85 terms, p. 86 like terms, p. 86 coefficient, p. 86 constant term, p. 86 coordinate plane, p. 91

x-axis, p. 91 y-axis, p. 91 origin, p. 91 quadrant, p. 91 ordered pair, p. 91 x-coordinate, p. 91 y-coordinate, p. 91

Vocabulary Review

- **1.** How many numbers have an absolute value of 15? List them.
- **2.** Copy the expression. List any like terms and coefficients.

$$-6y + 8 - 7x + 17x - 21y$$

3. How many quadrants are in a coordinate plane? Draw a coordinate plane and label each quadrant.

Copy and complete the statement.

- **4.** A point in a coordinate plane is represented by a(n) ?.
- **5.** A(n) ? is formed by the intersection of a horizontal number line and a vertical number line.
- **6.** The <u>?</u> is the sum of the values divided by the number of values.

Review Questions

Order the integers from least to greatest. (Lesson 2.1)

Write the opposite and the absolute value of the integer. (Lesson 2.1)

Find the sum or difference. (Lessons 2.2, 2.3)

13.
$$-81 + (-91)$$

15.
$$-324 + 500$$

16.
$$-468 + (-196)$$

Find the product. (Lesson 2.4)

21.
$$-6(9)$$

23.
$$-9(-23)(0)$$

Review Questions

Evaluate the expression when x = -6, y = -4, and z = -8. (Lesson 2.4)

26.
$$9z - 2x$$

27.
$$11y - 2xz$$

28.
$$2x + 3yz$$

29. Hot Air Balloon A hot air balloon at a height of 110 feet rises for 6 minutes at a rate of 18 feet per minute, then drops 22 feet per minute for 3 minutes. What is the height of the balloon? (Lesson 2.4)

Find the quotient. (Lesson 2.5)

30.
$$\frac{-26}{2}$$

31.
$$\frac{-98}{-7}$$

32.
$$\frac{-120}{-15}$$

33.
$$\frac{63}{-7}$$

Find the mean of the data. (Lesson 2.5)

Evaluate the expression using mental math. Name the property or properties used. (Lesson 2.6)

37.
$$(-45 + 97) - (-45)$$

38.
$$(-28 - 95 + 85) + (-62)$$

40.
$$(-15 \cdot 5) \cdot (-20)$$

40.
$$(-15 \cdot 5) \cdot (-20)$$
 41. $[-54 \cdot (-56)] \cdot 0 \cdot (-17)$

Use the distributive property to simplify the expression. (Lesson 2.7)

42.
$$5(12x - 20)$$

43.
$$7(9 + 11y)$$

44.
$$4(25z - 30)$$

Simplify the expression by combining like terms. (Lesson 2.7)

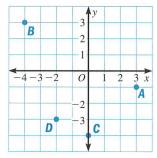
45.
$$14x - 3y - 7x + y$$

46.
$$4x - 11y + 2(1 - x)$$

47. Baseball Caps Use mental math and the distributive property to find the total price of 3 baseball caps that cost \$12.90 each. (Lesson 2.7)

Give the coordinates of the point. (Lesson 2.8)

49. B





52. Geometry Plot, label, and connect in order the following points in a coordinate plane. Find the perimeter of the figure. (Lesson 2.8)

$$A(0, 7), B(4, 7), C(4, 4), D(8, 4), E(8, 0), F(0, 0)$$

Chapter Test

Order the integers from least to greatest.

1.
$$-9$$
, 8, 14, -11 , 0, -1

Find the number.

3. Find
$$|x|$$
 when x is 2 and when x is -4

3. Find
$$|x|$$
 when x is 2 and when x is -4 .

4. Find $-(-x)$ when x is -1 and when x is 7 .

Find the sum or difference.

5.
$$17 + (-9)$$

6.
$$-8 + (-14)$$
 7. $-2 + (-21)$ **8.** $-33 + 26$

7.
$$-2 + (-21)^{\circ}$$

8.
$$-33 + 26$$

10.
$$-4 - 17$$

Find the product or the quotient.

13.
$$-5(14)$$

15.
$$\frac{-152}{-19}$$

16.
$$\frac{-132}{6}$$

17. Temperature The following temperatures were taken during a week in December in Nome, Alaska. What is the mean temperature to the nearest degree? $-5^{\circ}F$, $-8^{\circ}F$, $-13^{\circ}F$, $-16^{\circ}F$, $-8^{\circ}F$, $11^{\circ}F$, $0^{\circ}F$

Evaluate the expression when
$$a = 3$$
, $b = -15$, and $c = 15$.

18.
$$\frac{c}{-a}$$

19.
$$\frac{6b}{2c}$$

20.
$$\frac{c}{-5a}$$

21.
$$\frac{b^2}{a^2}$$

22. Groceries You are purchasing a loaf of bread for \$2.16, a box of cereal for \$3.25, and 2 cans of soup for \$.42 each. Write an expression to find the total cost. Then evaluate your expression.

Simplify the expression by combining like terms.

23.
$$3x + 4 - x + 1$$

24.
$$2x - 3y + 5x - (-9y)$$

25.
$$2(9x - 22y) + 4x$$

Plot the point in a coordinate plane and describe its location.

30. Critical Thinking Fill in the table for the equation y = |x|. Then plot the points on a coordinate plane. Describe the shape of the graph.

X	-3	-2	-1	0	1	2	3
y	?	?	?	?	?	?	?

Chapter Standardized Test

Test-Taking Strategy If you are unsure of an answer, try to eliminate answers that you know are wrong.

1. Which of the following shows the integers in order from least to greatest?

A.
$$-7$$
, 11, 5, -6 , 0

A.
$$-7$$
, 11, 5, -6 , 0 **B.** 0, -7 , -6 , 5, 11

C.
$$-7$$
, -6 , 0 , 5 , 11 **D.** 0 , -6 , 5 , -7 , 11

D.
$$0, -6, 5, -7, 11$$

2. Which of the following is equal to -10?

F.
$$-|-10|$$

G.
$$-(-10)$$

I.
$$|-10|$$

3. Which numerical expression represents the verbal expression "the absolute value of the opposite of negative 6"?

A.
$$-(-|6|)$$

B.
$$-(-6)$$

C.
$$|-(-6)|$$
 D. $-|-6|$

D.
$$-|-6|$$

4. What is the value of -21 + (-15) - 32?

G.
$$-24$$

H.
$$-4$$

5. If the temperature is -6° Fahrenheit and it changes by -12° Fahrenheit, what is the new temperature?

A.
$$-18^{\circ}$$
F

B.
$$-6^{\circ}$$
F

6. What is the value of 20 - x - (-y) when x = 9 and y = -6?

7. Which product is *not* equal to -336?

A.
$$7(-8)(2)(-3)$$

B.
$$56(-2)(3)$$

C.
$$3(-7)(8)(2)$$

D.
$$-8(-7)(-3)(2)$$

8. Evaluate -4ac + bc - 2a when a = 6, b = -2, and c = -3.

G.
$$-54$$

9. What is the mean of the following set of data?

$$-7, 6, -1, 11, -8, -9, 0, -8$$

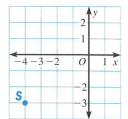
A.
$$-4$$

10. Which ordered pair represents point S in the coordinate plane?

F.
$$(3, -4)$$

G.
$$(-3, 4)$$

$$\mathbf{I}$$
. $(4, -3)$



Short Response

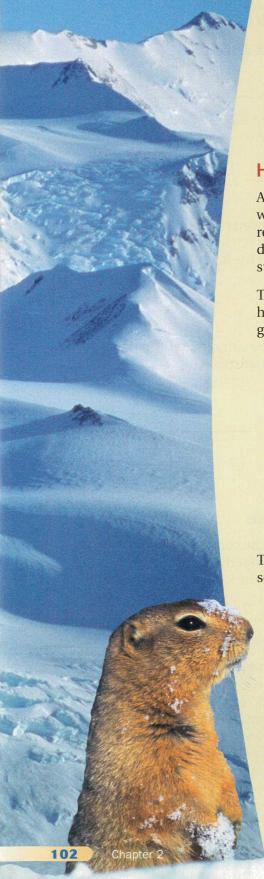
11. Explain how the commutative and associative properties of addition can help you find the sum using mental math.

$$(57 + 24) + (36 + 83)$$

Extended Response

12. You are planning a field trip to a museum. The bus ride will cost \$250. Admission is \$8 and lunch is \$5 for each person. Write two expressions to find the total cost for x students. What would the cost be if 20 students went on the trip? 25 students? Did you use the distributive property to find your answer? Explain why or why not.

EXPLORING MATH IN SCIENCE

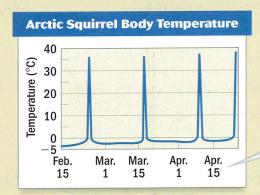


Supercool Squirrels

Hibernating in the Arctic

Animals living in very cold climates may hibernate to survive during winter. Hibernating animals have a decreased heart rate and decreased respiratory rate. The body temperature of a hibernating animal also drops significantly, often to just a few degrees higher than the surrounding temperature.

The arctic ground squirrel's body temperature drops below 0°C during hibernation. The graph below shows the body temperature of an arctic ground squirrel during the last few months of its hibernation.



The squirrel hibernates from early September until late April.

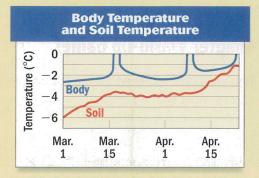
The peaks in the graph represent short periods of time during which the squirrel wakes up from hibernation and warms itself up.

- **1.** Estimate the difference of the squirrel's highest and lowest body temperatures for the period shown on the graph.
- **2.** Which is the best estimate for the longest period during which the squirrel's body temperature was above zero? below zero?
 - A. 1 hour
- B. 1 day
- C. 5 days
- **D.** 20 days
- **3.** A squirrel's approximate body temperatures for several days in March were –3°C, –1°C, 36°C, 6°C, –2°C, and –3°C. What is the mean of the temperatures?
- **4. Critical Thinking** Do you think your answer from Exercise 3 is a good representation of the squirrel's usual body temperature during the period of time shown by the graph? Explain your reasoning.

Soil Temperature

The graph below displays the squirrel body temperature data for March and April. It has a smaller temperature range, so waking periods are not fully displayed in the graph.

It also shows the soil temperature at 1 meter below ground, the depth at which the squirrel usually digs its burrow.



- **5.** What is the difference of body temperature and soil temperature on March 1? What is the difference of these two temperatures on April 15th?
- **6. Critical Thinking** A scientist claims that a hibernating animal's body temperature is always within 1 or 2 degrees of the surrounding temperature. Discuss whether this claim seems reasonable for the arctic ground squirrel, based on the data given.

Project IDEAS

- Report Find out more about the arctic ground squirrel. In what areas does it live? What foods does it eat? What are its predators? Present your findings to the class.
- Research Some reptiles and amphibians, such as the painted turtle and the wood frog, can survive subzero temperatures.
 Find out more about one of these animals. How low can its body temperature go? What mechanisms does it use to avoid freezing?
 Present your findings to the class.
- Career Find out more about the work that biologists do. What are some topics or areas in which a biologist might specialize? Present your findings to the class.

Project Support



Solving Equationsand Inequalities

BEFORE

In previous chapters you've...

- Evaluated expressions
- Solved word problems

Now

In Chapter 3 you'll study...

- · Solving one-step equations
- · Solving two-step equations
- Writing equations
- · Formulas for area and perimeter
- · Writing and solving inequalities
- · Real life modeling

WHY?

So you can solve real-world problems about...

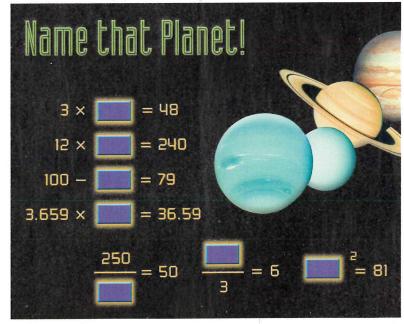
- · sea lions, p. 112
- cartoonists, p. 121
- biplane rides, p. 143
- DJs, p. 149

Internet Preview CLASSZONE.COM

- eEdition Plus Online
- eWorkbook Plus Online
- eTutorial Plus Online
- State Test Practice
- More Examples

Chapter Warm-Up Games

Review Skills you need for this chapter in these quick games. Work with a partner.





Key Skill:Solving mental math equations

- · Use mental math to find the missing number.
- Match each answer with the corresponding letter of the alphabet.
 (1 = A, 2 = B, etc.)
- Rearrange the letters to find the name of a planet.