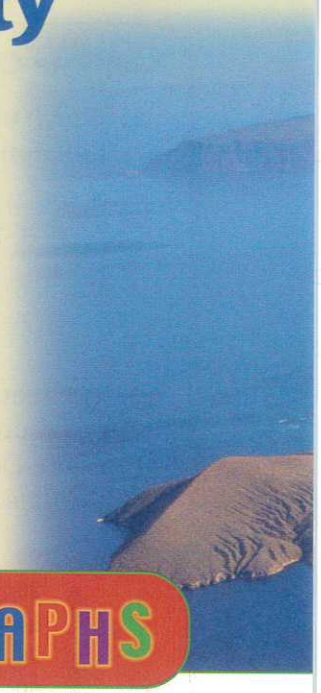
CHAPTER 12

Data Analysis and Probability



BEFORE

In previous chapters you've...

- Found the median and range of a data set
- Found probabilities of events

NOW

In Chapter 12 you'll study...

- Interpreting data displays
- Using permutations and combinations to count possibilities
- Finding the probability of independent and dependent events

WHY?

So you can solve real-world problems about...

- electricity, p. 599
- skateboards, p. 619
- poetry, p. 624
- archery, p. 636

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Chapter Warm-Up Game

Review skills you need for this chapter in this quick game.

Key Skill:
Interpreting bar and circle graphs

GALAPAGOS GRAPHS

HOW TO PLAY

1. **USE** the data displays to answer each question. Record the letter for each correct answer.

   About how many times larger is Santa Cruz than San Cristobal?
   A. 2 **B.** 3 **C.** 4

   What is the area of Isabela?
   D. 1680 mi^2 **E.** 1771 mi^2 **F.** 1800 mi^2

   What percent of the land vertebrate species in the Galapagos Islands are reptiles?
   G. 10% **H.** 26% **I.** 30%

2. **WRITE** the letters of the correct answers to the questions in order. Find the number that corresponds to the letter. Put all three numbers together in the order of the answers. This will tell you the age of the oldest Galapagos tortoise on record.

   A. 1 **B.** 2 **C.** 0 **D.** 9 **E.** 5
   F. 2 **G.** 7 **H.** 2 **I.** 5

Chapter Standardized Test

Test-Taking Strategy **When you check your answers, try to use a method other than the one you originally used, to avoid repeating the same mistake.**

Multiple Choice

1. Which of the following relations is *not* a function?

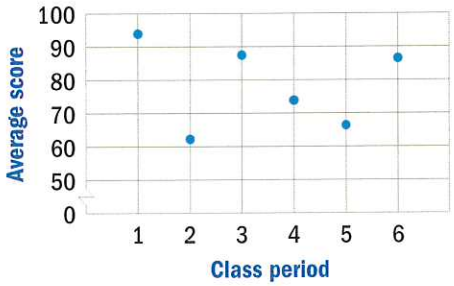
A. (5, 6), (6, 5), (−3, 7), (2, 6)

B. (−5, 6), (5, 6), (−3, 7), (2, 4)

C. (5, 6), (7, 7), (1, −1), (5, 0)

D. (3, 2), (−3, −2), (2, 3), (−2, −3)

2. The scatter plot below shows the average test score of each class on a recent English exam. Describe the relationship shown in the data.



F. positive

G. negative

H. none

I. not enough information

3. Which of the ordered pairs is a solution of the equation $y = 4x - 5$?

A. (−2, −13)

B. (0, 5)

C. (3, 17)

D. (7, 3)

4. What is the slope of the line passing through points (−3, 2) and (4, −5)?

F. −3

G. −1

H. $\frac{1}{3}$

I. 1

5. Which is the slope-intercept form of $9x - 3y = 12$?

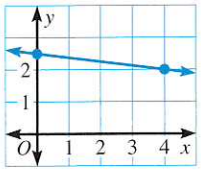
A. $-3y = -9x + 12$

B. $x = \frac{1}{3}y + \frac{4}{3}$

C. $y = 3x + 4$

D. $y = 3x - 4$

6. Which equation is represented by the graph?



F. $y = -\frac{1}{8}x + 2.5$

G. $y = \frac{1}{8}x + 2.5$

H. $y = -8x + 2.5$

I. $y = 8x + 2.5$

7. What is the value for y in the function table representing $y = x - 7$, when x is 7?

x	3	5	7	9
y	−4	−2	?	2

A. −1

B. 0

C. 1

D. 4

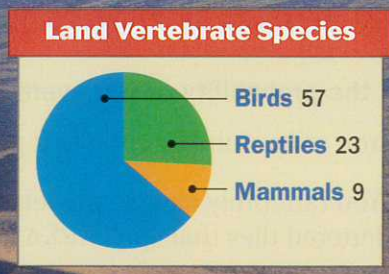
Short Response

8. Graph the inequalities $x + y \leq 5$, $x \geq 0$, and $y \geq 0$. Find the area of the solution region.

Extended Response

9. Your class needs to raise $120 selling hats and T-shirts. The equation $6x + 5y = 120$, where x is the number of hats sold and y is the number of T-shirts sold, represents the situation. What are the intercepts of the equation? What do they mean in this context?

Great Frigatebird



Giant Galapagos Tortoise

Galapagos Sea Lion

Stop *and* Think

1. **Writing** Describe one way the circle graph would change if you included the ocean life of the Galapagos Islands.
2. **Critical Thinking** Is the area of Isabela greater than the total area of the next four largest islands? Explain.

CHAPTER 12

Getting Ready to Learn

Review What You Need to Know

Using Vocabulary **Copy and complete using a review word.**

1. When you flip a coin, heads and tails are the two possible ? .
2. Find the sum of the values of a set of data and then divide by the number of data values to find the ? of the data.
3. The ? is a measure of how likely it is that the event will occur.

Find the mean and the median of the data set. *(p. 257)*

4. 23, 27, 13, 24, 19, 21, 25, 25, 12
5. 0.2, 0.35, 1.33, 1.32, 0.05, 0.5

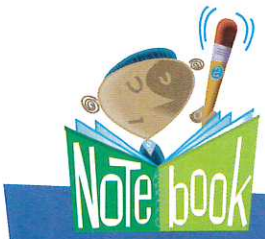
Find the probability of the event. *(p. 354)*

6. You roll a number cube and get a number greater than 4.
7. You randomly choose the letter A from a bag holding the eight lettered tiles that spell ARKANSAS.

Word Watch

Review Words

data, p. 5
mean, p. 257
median, p. 257
range, p. 258
outcome, p. 354
probability of an event, p. 354



You should include material that appears on a notebook like this in your own notes.

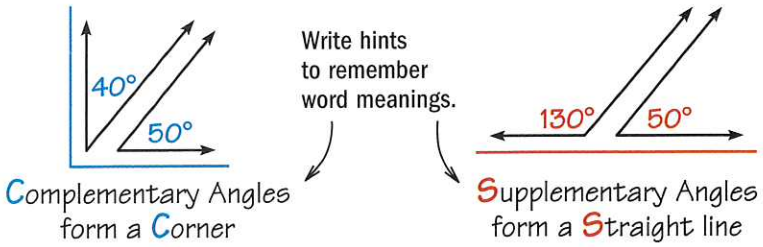
Know How to Take Notes

Contrasting Terms When words have similar meanings, you should emphasize their differences in your notes.

Pairs of Angles

Complementary Angles: The sum of the angle measures is 90°.

Supplementary Angles: The sum of the angle measures is 180°.



Complementary Angles form a Corner

Supplementary Angles form a Straight line

In Lesson 12.6, you should note the difference between combinations and permutations.

LESSON 12.1

Stem-and-Leaf Plots

BEFORE You organized data using bar graphs and histograms.

NOW You will make and interpret stem-and-leaf plots.

WHY? So you can analyze waiting times at a restaurant, as in Ex. 14.

Word Watch

stem-and-leaf plot, p. 597

In the Real World

Track Hurdlers entering the 200 meter hurdles at a track meet were ranked according to their qualifying times, in seconds, shown below.

28.6, 29.2, 28.1, 27.5, 29.8, 28.7, 30.2, 29.3, 28.3, 28.9, 29.9, 28.4

How can the data be displayed to show the distribution of the times?

A **stem-and-leaf plot** is a data display that helps you see how data are distributed. You can use a stem-and-leaf plot to order data.

EXAMPLE 1 Making a Stem-and-Leaf Plot

You can display the hurdlers' times given above in a stem-and-leaf plot.

1. The times range from 27.5 to 30.2. Let the **stems** be the digits in the tens' and ones' places. Let the **leaves** be the tenths' digits.
2. Write the stems first. Then record each time by writing its tenths' digit on the same line as its corresponding stem. Include a key that shows what the stems and leaves represent.
3. Make an ordered stem-and-leaf plot.

Unordered Plot

27	5
28	6 1 7 3 9 4
29	2 8 3 9
30	2

Key: 27|5 = 27.5

Ordered Plot

27	5
28	1 3 4 6 7 9
29	2 3 8 9
30	2

Key: 27|5 = 27.5

The leaves for each stem are listed in order from least to greatest.

Your turn now **Make an ordered stem-and-leaf plot of the data.**

1. Video game prices: \$40, \$15, \$10, \$19, \$12, \$24, \$15, \$39, \$51, \$50, \$35, \$20, \$47, \$36, \$30, \$25, \$27, \$29, \$24, \$43, \$29

HELP with Solving

In a stem-and-leaf plot, a stem can be one or more digits. A leaf is usually a single digit.

EXAMPLE 2 Interpreting a Stem-and-Leaf Plot

Biology The stem-and-leaf plot at the right shows the lengths, in millimeters, of young fish in a tank. Use the stem-and-leaf plot to describe the data. What interval includes the most lengths?

4	9
5	
6	1 2 4 7 8 8
7	6 8
8	4

Key: 6|8 = 68

Solution

The longest fish is 84 mm and the shortest fish is 49 mm, so the range of lengths is 35 mm. Most of the lengths are in the 60–69 interval.

Double Stem-and-Leaf Plots A double stem-and-leaf plot can be used to compare two sets of data. You read to the left of the stems for one set of data and to the right for the other.

EXAMPLE 3 Making a Double Stem-and-Leaf Plot

Test Scores The data below show the test scores for Beth's class and Marisa's class. Overall, which class had the better test scores?

Beth's class: 95, 86, 79, 79, 58, 68, 90, 63, 71, 81, 82, 94, 64, 76, 77, 79, 83, 91, 83, 68, 74, 71

Marisa's class: 95, 73, 76, 84, 84, 89, 67, 82, 88, 86, 93, 97, 96, 84, 60, 75, 91, 87, 89, 86, 76, 93

Solution

You can use a double stem-and-leaf plot to compare the test scores.

Beth's Class		Marisa's Class
8	5	
8 8 4 3	6	0 7
9 9 9 7 6 4 1 1	7	3 5 6 6
6 3 3 2 1	8	2 4 4 4 6 6 7 8 9 9
5 4 1 0	9	1 3 3 5 6 7

Key: 0|9|1 represents 90 and 91.

ANSWER Marisa's class; it had more scores in the eighties and nineties.

Your turn now **Complete the following exercises.**

2. Make an ordered double stem-and-leaf plot to compare the lengths, in minutes, of the last 10 phone calls made by two friends.

Kenyon: 12, 8, 17, 5, 23, 29, 21, 34, 16, 28
Jason: 31, 28, 7, 5, 11, 5, 13, 16, 11, 24

3. In general, who made longer calls, Kenyon or Jason?

12.1 Exercises

More Practice, p. 738

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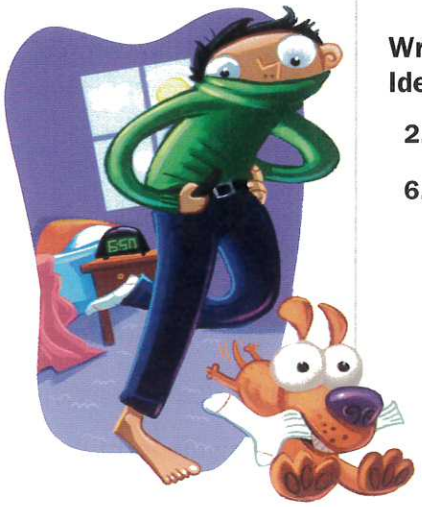
Getting Ready to Practice

1. Vocabulary Copy and complete: The key for a stem-and-leaf plot says 7|4 = 74. In the plot, 7 is the ? and 4 is the ? .



Write the number as it would appear in a stem-and-leaf plot. Identify the stem and the leaf.

2. 80 **3.** 117 **4.** 12.9 **5.** 4.6

6. Guided Problem Solving The data show the times, in minutes, it takes ten students to get ready for school. Make an ordered stem-and-leaf plot of the data. What interval includes the most time values?

25, 10, 25, 15, 30, 18, 35, 40, 28, 20

(1) Identify the range of the data.

(2) Make the stem-and-leaf plot. Include a key.

(3) Use the plot to find where most of the times fall.

Practice and Problem Solving

Make an ordered stem-and-leaf plot of the data. Identify the interval that includes the most data values.

7. 45, 48, 65, 50, 67, 82, 74, 63

8. 33, 12, 8, 14, 35, 9, 26, 37, 4, 6

9. 108, 95, 89, 112, 109, 94, 103

10. 461, 492, 439, 467, 501, 485

11. 20.2, 22.6, 18.3, 18.7, 22.5, 18.1

12. 5.1, 4.0, 5.3, 3.2, 5.7, 6.9, 5.3

13. Writing Explain how you can use an ordered stem-and-leaf plot to find the median value of a set of data.

14. Restaurants The stem-and-leaf plot shows the average waiting times, in minutes, to be seated for fifteen restaurants. What are the shortest and longest waiting times? Which interval has the fewest number of waiting times?

0	5 6 9
1	2 5 5
2	0 0 5 8
3	2 8
4	0 5 5

Key: 2|5 = 25

15. Electricity The data show the amounts, in dollars, of a family's electric bills for twelve months. Make an ordered stem-and-leaf plot. What is the range? Are the bills more often greater or less than $60?

95, 58, 47, 78, 43, 65, 84, 72, 55, 84, 96, 59

HELP with Homework

Example	Exercises
1	7–12
2	13–15, 18
3	16–17, 19

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Make an ordered double stem-and-leaf plot of the two sets of data.

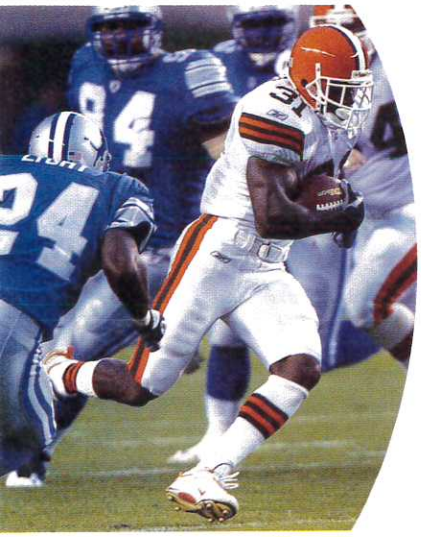
16. Set A: 16, 19, 8, 22, 18, 20, 32, 5

Set B: 12, 8, 25, 42, 31, 15, 16, 9

17. Set C: 102, 98, 111, 70, 118, 92, 77

Set D: 115, 88, 87, 102, 65, 95, 93

18. Critical Thinking Can you make a stem-and-leaf plot from a frequency table? Why or why not?



19. Football The total points that the Cleveland Browns scored in each game of a recent season are given below. Red numbers represent wins and blue numbers represent losses. Make an ordered double stem-and-leaf plot of the data. Describe the relationship between points scored and the outcome of the game.

6, 24, 23, 20, 14, 24, 21, 12, 27, 18, 15, 16, 10, 7, 41, 7

Extended Problem Solving **In Exercises 20–22, use the data below, which show the average monthly temperatures in degrees Fahrenheit (°F) for Los Angeles, California.**

56.8, 57.6, 58.0, 60.1, 62.7, 65.7, 69.1, 70.5, 69.9, 66.8, 61.6, 56.9

20. Plot Make an ordered stem-and-leaf plot of the data. What is the range?

21. Convert Convert the data to degrees Celsius (°C) using the formula $C = \frac{5}{9}(F - 32)$. Round to the nearest tenth of a degree.

22. Compare Make an ordered stem-and-leaf plot of the converted data. Compare the two plots. In what ways are they different? Explain.

Mixed Review

23. Find the mean, median, mode(s), and range of the data. *(Lesson 5.8)*

−22, 14, 12, 6, −10, 14, 20, 16, −7, −5, 6, −2

24. Find the slope and y-intercept of the line $7x + 4y = 24$. *(Lesson 11.7)*

Test-Taking Practice

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The stem-and-leaf plot shows the ages of people at a birthday party. Use the plot to answer Exercises 25 and 26.

0	6
1	3 7 9
2	0 1 4 5
3	
4	7 9
5	0 1 2 2 2

Key: 4|7 = 47

25. Multiple Choice What is the age of the oldest person?

A. 52 **B.** 59 **C.** 60 **D.** 79

26. Multiple Choice What is the median age?

F. 52 **G.** 35 **H.** 30 **I.** 25

LESSON 12.2

Box-and-Whisker Plots

BEFORE You found the median and range of a data set.

NOW You will make and interpret box-and-whisker plots.

WHY? So you can analyze camera prices, as in Ex. 12.

In the Real World

Bridges The lengths, in meters, of the world's ten longest suspension bridges are listed below. How can you display these data to show how the lengths are distributed?

1280	1118	1990	1074	1298
1067	1624	1158	1090	1410

A **box-and-whisker plot** is a data display that organizes data values into four parts. Ordered data are divided into lower and upper halves by the median. The **lower quartile** is the median of the lower half of the data set. The **upper quartile** is the median of the upper half of the data set.

The **lower extreme** is the least data value and the **upper extreme** is the greatest data value.

Word Watch

box-and-whisker plot, p. 601
lower quartile, p. 601
upper quartile, p. 601
lower extreme, p. 601
upper extreme, p. 601

EXAMPLE 1 Making a Box-and-Whisker Plot

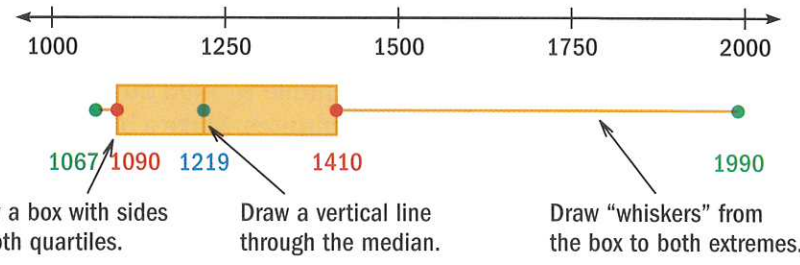
To display the bridge lengths above in a box-and-whisker plot, first order the data to find the median and the quartiles.

$$\text{Median} = \frac{1158 + 1280}{2} = 1219$$

Lower half: 1067 1074 **1090** 1118 1158 — Upper half: 1280 1298 **1410** 1624 1990

Lower quartile = 1090 — Upper quartile = 1410

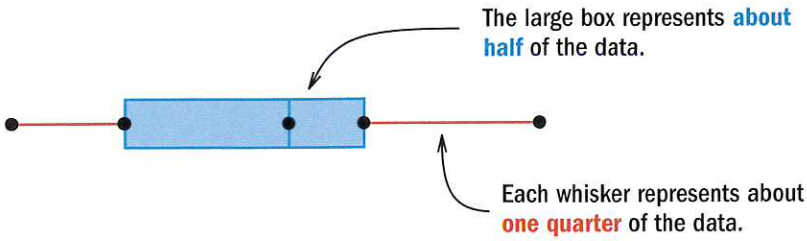
Plot these values below a number line that includes the extremes.



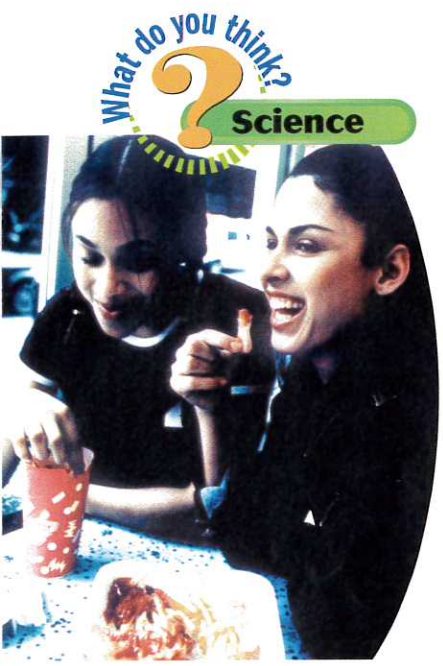
HELP with Solving

If a data set has an odd number of values, then the median is not included in either half of the data when determining the quartile values. For help with finding a median, see p. 257.

Interpreting a Box-and-Whisker Plot A box-and-whisker plot helps to show how varied, or spread out, the data are. The points divide the data into four parts. Each part represents about one quarter of the data.



You can use box-and-whisker plots to compare two or more data sets.

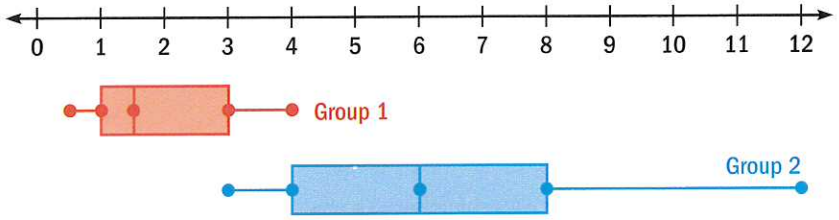


Food Science

It takes about 24 pounds of tomatoes to make 7 pints of ketchup. At this rate, how many pounds of tomatoes are used to make a 2 pint bottle of ketchup?

EXAMPLE 2 Interpreting Box-and-Whisker Plots

Food Science You are testing whether a fertilizer helps tomato plants grow. You give fertilizer to the plants in Group 2, but not to Group 1. The box-and-whisker plots show how much the plants grew, in centimeters, for each group of plants after two weeks.



a. About what fraction of the unfertilized plants grew as much as any of the fertilized plants?

b. About what fraction of the fertilized plants grew 4 to 8 centimeters?

Solution

a. Notice that the right whisker for Group 1 overlaps the left whisker for Group 2. So about one quarter of the unfertilized plants grew as much as the any of the fertilized plants.

b. The large box in the plot for Group 2 ranges from 4 to 8, so about one half of the fertilized plants grew 4 to 8 centimeters.

Your turn now **Use a box-and-whisker plot.**

1. Ming worked out for 34, 27, 26, 15, 24, 21, 30, 23, 24, and 35 minutes. Chantelle worked out for 26, 33, 36, 21, 41, 36, 29, 25, 34, and 35 minutes. Make a box-and-whisker plot of the data for each person.

2. Who usually works out longer? Explain.

3. About how often does each person work out for 25–35 minutes?

12.2 Exercises

More Practice, p. 738

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Getting Ready to Practice

1. **Vocabulary** The median of the lower half of a data set is the ? and the median of the upper half of a data set is the ?.

2. **Guided Problem Solving** You had the following scores while playing a math game: 306, 211, 235, 197, 351, 141, and 227. Make a box-and-whisker plot of your scores. Then predict your next score.

   (1) Find the range and draw a number line.

   (2) Find the median, quartiles, and extremes.

   (3) Draw the box-and-whisker plot.

   (4) Predict a range for your next score. Explain your reasoning.

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	3–13
2	12–15

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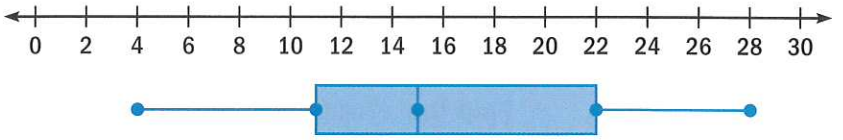
In Exercises 3–5, make a box-and-whisker plot of the data.

3. \$67, \$53, \$41, \$33, \$52, \$28, \$70, \$56

4. 327 ft, 419 ft, 9 ft, 299 ft, 111 ft, 0 ft

5. 26 m, 389 m, 878 m, 144 m, 515 m, 404 m

The box-and-whisker plot shows the lengths, in inches, of the jumps of frogs in a frog-jumping contest. Estimate the following values.

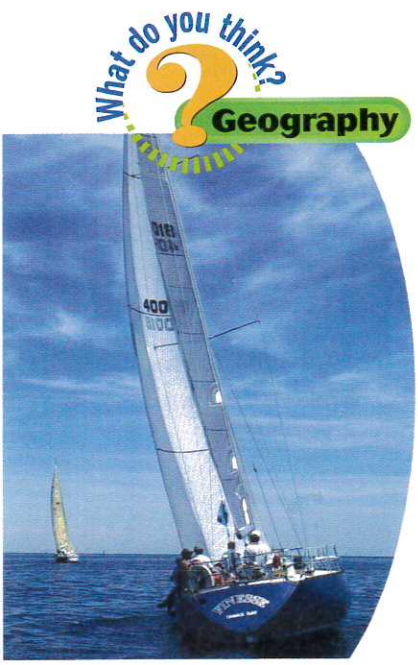


6. range
7. median
8. lower quartile
9. upper quartile
10. lower extreme
11. upper extreme

12. **Camera Prices** The prices of several cameras are \$179.99, \$329.99, \$229.99, \$284.99, \$399.99, \$379.99, \$299.99, \$259.99, and \$259.99. Organize the list of prices from least to greatest. Then make a box-and-whisker plot of the data. What conclusions can you make?

13. **Pumpkins** The weights, in pounds, of 10 giant pumpkins were 853, 811.5, 785, 1020, 826.5, 789, 838, 810, 731, and 822.5. Make a box and-whisker plot of the data. Describe what the plot shows.

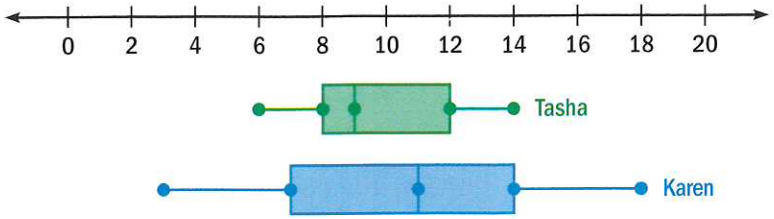
14. **Critical Thinking** Explain how making a stem-and-leaf plot can help you to make a box-and-whisker plot.



Lake Area

Three of the world's 10 largest lakes are Great Lakes. The 6 quadrillion gallons in the Great Lakes are 18% of the world's fresh water supply. One quadrillion is equal to one million billions. How much fresh water does the world have?

15. Basketball The box-and-whisker plots show the points scored per game for two players. What conclusions can you make about the players' performances? Which player is more consistent? Explain.



Lake Area **In Exercises 16 and 17, use the areas, in square kilometers, of the world's ten largest lakes: 371,000, 84,500, 64,500, 63,500, 62,940, 58,020, 32,000, 31,500, 31,400, and 28,400.**

An *outlier* is a data value that is much less or much greater than most of the other values in the data set.

16. Make a box-and-whisker plot of the data. Which value is an outlier?

17. Remove the outlier and then make another box-and-whisker plot. Describe how an outlier affects a box-and-whisker plot.

18. Challenge Change one value in the data set 3, 4, 5, 7, 9, 11, 13, 15, 17, 18, 21 so that the median of the set is 13, the lower quartile is 7, and the upper quartile is 17. Explain how you got your answer.

Mixed Review

19. The table shows the height, in inches, of several players on a soccer team. Make a bar graph of the data. *(Lesson 1.1)*

Name	Ally	Nate	Bob	Inez	Dan	Lisa
Height	68	66	73	66	60	62

Find the *x*-intercept, *y*-intercept, and slope of the graph of the equation. Then graph the line. *(Lessons 11.5, 11.6)*

20. $y = 24$ **21.** $3x - 5y = 30$ **22.** $x = -4$

Test-Taking Practice

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23. Extended Response The masses, in grams, of 10 samples from bolt factories A and B are shown. All bolts should be 198.5–202 grams.

Factory A: 199, 201, 200, 198.5, 200.5, 202, 201, 200.8, 200.9, 198.5

Factory B: 201, 200.4, 203, 200.8, 201, 203.4, 200.6, 201, 200.9, 203.1

Make box-and-whisker plots comparing the samples. Describe how well each factory makes bolts within the desired mass range, based on the samples. Then use the plots to compare the factories' performances.

LESSON 12.3

Using Data Displays

BEFORE You organized data using box-and-whisker plots.

NOW You will organize data using circle graphs and line graphs.

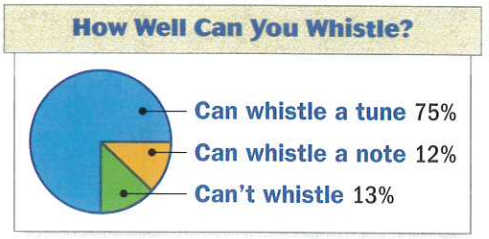
WHY? So you can represent the areas of boroughs of New York, as in Ex. 13.

Word Watch

circle graph, p. 605
line graph, p. 606

Circle Graphs A survey asked, "How well can you whistle?" The results are shown in the *circle graph* below. It shows that three out of four people can whistle a tune.

A **circle graph** represents data as sections of a circle. Each section can be labeled using a fraction, decimal, or percent. Because the graph represents all the data, the sum of the sections must equal 1, or 100%.



To make a circle graph, find the angle measure to the nearest degree that represents each data value's portion of the whole. The sum of all the angle measures must equal 360°, the number of degrees in a circle.

EXAMPLE 1 Making a Circle Graph

E-mail A survey asked, "How often do you check your e-mail?" Of the 100 people asked, 4 answered *less than weekly*, 23 answered *weekly*, and 73 answered *every day*. You can display the data in a circle graph.

HELP with Solving

In Example 1, you can draw the 14° and 83° angles first. Then the remaining section of the circle will have a measure of 263°.

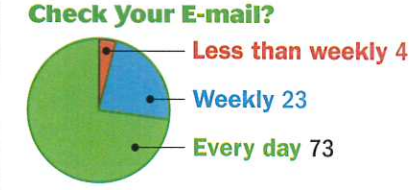
(1) Use a proportion to find the number of degrees to use to represent each response as a section in a circle graph.

Less than weekly	Weekly	Every day
$\frac{4}{100} = \frac{a}{360°}$	$\frac{23}{100} = \frac{b}{360°}$	$\frac{73}{100} = \frac{c}{360°}$
$a = 14.4° \approx \mathbf{14°}$	$b = 82.8° \approx \mathbf{83°}$	$c = 262.8° \approx \mathbf{263°}$

(2) Draw a circle.

(3) Use a protractor to draw the first angle measure. Then label the section.

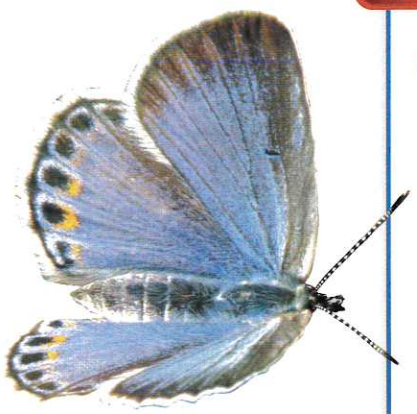
(4) Draw and label remaining sections. Include a title.



ANSWER The graph shows that the majority of people check their e-mail every day.

Line Graphs A **line graph** represents data that change over time.

EXAMPLE 2 Making a Line Graph



Karner Blue Butterfly, endangered since 1992

Environment **The table shows the number of insect species on the United States endangered species list. Make a line graph of the data.**

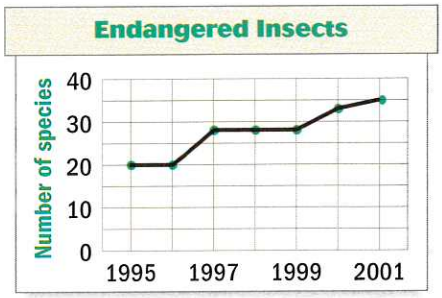
Year	1995	1996	1997	1998	1999	2000	2001
Number	20	20	28	28	28	33	35

(1) Draw and label the horizontal and vertical scales.

(2) Plot a point for each data pair.

(3) Draw line segments to connect the points.

(4) The graph shows an increase over time.

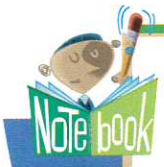


Your turn now **Use the table of polling data.**

1. Make a circle graph of the data for Week 1. What does the graph show?
2. Make a line graph of the data for Ben. What does the graph show?

Who Will You Vote For?			
Week	1	2	3
Ben	55%	50%	40%
Alice	45%	50%	60%

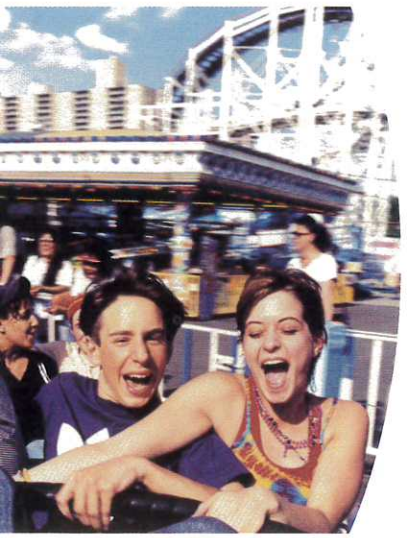
Using appropriate data displays helps you make meaningful conclusions.



Using Appropriate Data Displays

- Use a *circle graph* to represent data as parts of a whole.
- Use a *line graph* to display data over time.
- Use a *stem-and-leaf plot* to order a data set.
- Use a *box-and-whisker plot* to show the data's distribution in quarters, using the median, quartiles, and extremes.
- Use a *bar graph* to display data in distinct categories.
- Use a *histogram* to compare the frequencies of data that are grouped in equal intervals.

EXAMPLE 3 Choosing a Data Display



Choose an appropriate display for the data.

a. The table below shows the results of a survey that asked students if they are going away during summer vacation.

Response	Percent
Yes	48%
No	37%
Don't know	15%

b. The table below shows the results of a survey that asked students about ways they use the Internet.

Purpose	Percent
Research	62%
Shopping	34%
E-mail	45%
News	10%
Browsing	18%

Solution

a. The data add up to 100%, so a circle graph is appropriate.

b. The percents in the categories add up to more than 100%. An appropriate display for the data is a bar graph.

12.3 Exercises

More Practice, p. 738

INTERNET eWorkbook Plus CLASSZONE.COM

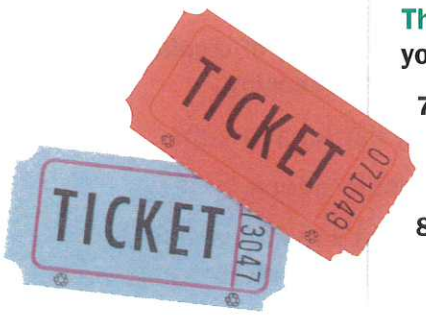
Getting Ready to Practice

Vocabulary **Copy and complete the statement.**

1. You can use a ? to display data as parts of a whole.
2. You can use a ? to display changes in a quantity over time.

Convert the value into an angle measure for display in a circle graph.

3. 31%
4. $\frac{3}{8}$
5. 14%
6. 27 out of 60



Theater **In Exercises 7 and 8, use the table showing attendance at your school play.**

7. Make a circle graph to display each attendance number as a fraction of total attendance at all four performances.
8. Tickets cost $7 each. Make a line graph that shows how much money was collected from each performance.

Play Attendance	
Friday	130
Saturday (2 P.M.)	231
Saturday (8 P.M.)	291
Sunday	185

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	9, 12–17
2	10–11, 15–17
3	15–17

Online Resources
CLASSZONE.COM
- More Examples
- eTutorial Plus

9. Snacking The table shows the results of a survey asking students to describe how often they snack. Represent the data in a circle graph.

How Often Do You Snack?	
Never	10%
Rarely	45%
Sometimes	35%
Often	10%

10. New Houses The table below shows the number, in thousands, of new single-family houses sold in the United States each year from 1997 to 2001. Make a line graph of the data. Describe how the data change over time.

Year	1997	1998	1999	2000	2001
Houses sold (thousands)	804	886	880	877	900

11. Music The table below shows the number of people, in millions, who attended symphony orchestra concerts in the United States each year from 1994 to 1998. Make a line graph of the data. What trend does the graph show?

Year	1994	1995	1996	1997	1998
Attendees (millions)	24.4	30.9	31.1	31.9	32.2

HELP with Solving

The sum of the rounded angle measures in Exercise 13 will not equal 360°. This is the result of *round-off error*. The sum of the unrounded values is 360°.

New York City **The map shows the land area, in square miles, of each of the five boroughs of New York City.**

12. Find the percent of the total area of New York City that each borough covers.

13. Represent the data using a circle graph.

14. Use the graph to compare the areas of the boroughs.

Critical Thinking **In Exercises 15–17, tell which type of display you would use for the data described. Explain your reasoning.**

15. You record the temperature at noon every day for a month.

16. You record the temperature at five different locations.

17. You record the high temperature every day for a month, and find how often the daily high temperature falls within each 10 degree temperature interval.

Car Color **In Exercises 18 and 19, use the table. It shows the percent of people who liked the given color for sports and compact cars.**

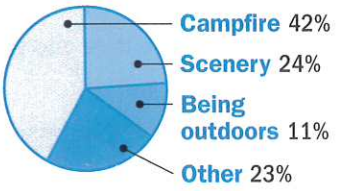
Color	Silver	Black	Dark blue	White	Dark green	Red	Other
2000	25.4%	14.5%	11.3%	9.8%	6.7%	12.7%	19.6%
2001	22.3%	14.4%	5.0%	11.4%	9.7%	15.8%	21.4%

18. Make a circle graph of the data for 2000 and for 2001.

19. From 2000 to 2001, which color had the largest percent increase? The largest percent decrease?

20. **Critical Thinking** Is it easier to answer Exercise 19 by comparing the graphs or by using the table? Explain.

21. **Camping** The data in the circle graph show the percent of people who chose each reason for camping. Can you display the data in a line graph? Why or why not? Can you display the data in a bar graph? Why or why not?



22. **Challenge** Describe a set of data that could be displayed in three different types of data display. Explain how to display it in each.

Mixed Review

Graph the equation using intercepts. *(Lesson 11.5)*

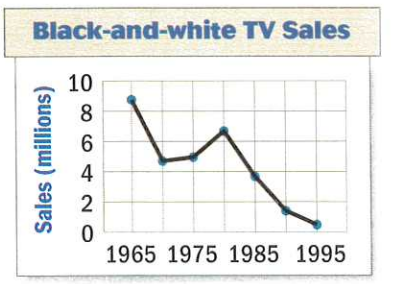
23. $5x + 2y = 10$ **24.** $8x - 3y = 24$ **25.** $9y - 18x = 27$

26. Make an ordered stem-and-leaf plot of the following data: 17, 10, 11, 15, 21, 34, 26, 16, 36, 24, 37, 20, 18, 31, 39, 29, 28. *(Lesson 12.1)*

Test-Taking Practice

INTERNET
State Test Practice
CLASSZONE.COM

27. **Multiple Choice** Use the line graph. It shows the numbers, in millions, of black-and-white TVs sold from 1965 to 1995. In which time period is the decrease the greatest?



A. 1965–1970 **B.** 1975–1980

C. 1985–1990 **D.** 1990–1995

28. **Short Response** You ask 100 people a yes or no question. The possible answers are *yes*, *no*, and *no answer*. What types of display would be appropriate for this type of data? Explain your reasoning.

12.3 Technology Activity

GRAPHING CALCULATOR

Making Data Displays

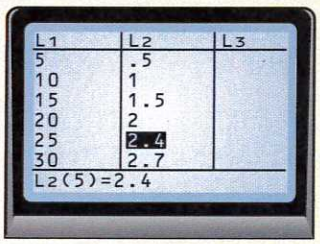
GOAL Use a graphing calculator to create data displays.

Example 1 **The table shows the cost, in dollars, of a phone call, based on the length, in minutes, of the call. Make a scatter plot or a line graph on your calculator.**

Minutes	Cost
5	0.5
10	1
15	1.5
20	2
25	2.4
30	2.7

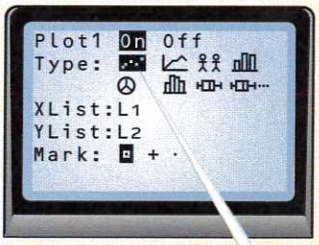
Solution

1 Enter the data into two lists.



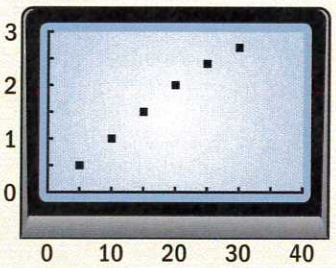
2 Choose maximums and minimums for the window.

3 Choose a display from the PLOT menu and set the Xlist and Ylist.



Choose for a scatter plot or for a line graph.

4 Press GRAPH to show the display you have chosen.



Your turn now **Make a scatter plot and a line graph of the data.**

1.

Year	1997	1998	1999	2000	2001
Rainfall (in.)	21.4	39.8	34.5	26.1	44.9

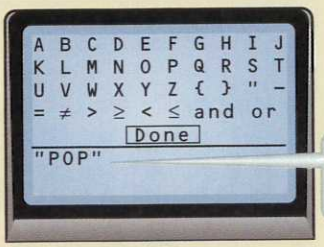
Example 2

The table shows the number of people out of 200 surveyed who prefer each type of music. You can use a graphing calculator to make a circle graph of the data.

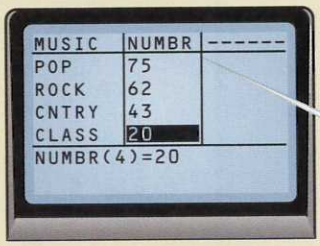
Pop	75
Rock	62
Country	43
Classical	20

Solution

1 Press LIST. Then use the TEXT menu to name a list and its categories. Use quotation marks for the first item, so the calculator recognizes that the list is *categorical* (contains words).

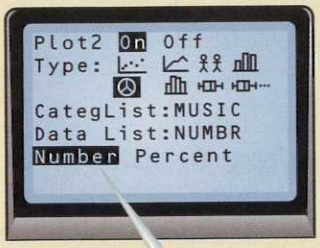


Use quotation marks for categorical data.



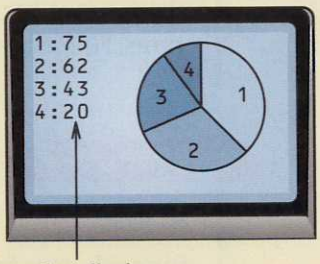
Enter the numerical data into a second list.

2 Use the PLOT menu to choose the two lists.



You can display the data as numbers or as percents.

3 Press GRAPH to display the circle graph.



number display

Your turn now **Make a circle graph of the survey data.**

2.

Did You Get Enough Sleep?	
Need more	541
Need less	167
Just right	282
Don't know	21

3.

The last movie I saw I watched...	
in a theater.	410
on a television.	483
on a computer.	21

CHAPTER 12 Special Topic

Misleading Graphs

GOAL Identify and analyze misleading graphs.

Word Watch

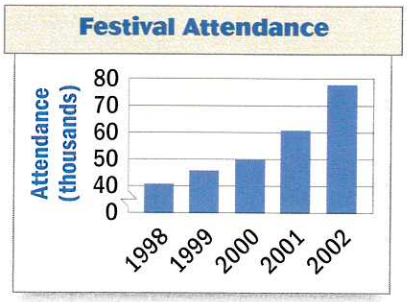
Review Words

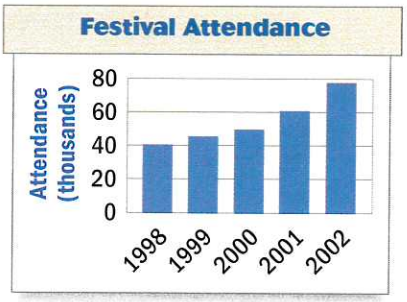
bar graph, p. 5
line graph, p. 606

When you analyze a graph to make conclusions based on the data displayed, it is important to be aware that the display may be misleading.

EXAMPLE 1 Identifying a Misleading Graph

Attendance Which of the bar graphs that show attendance at an annual rock festival from 1998 to 2002 could be misleading?





HELP with Solving

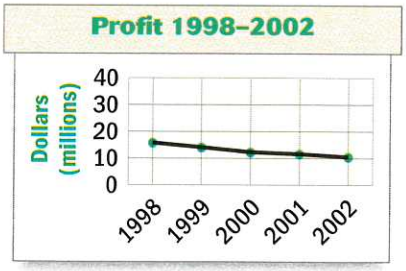
Remember that the broken axis symbol ⍭ indicates that some of the values in the axis have been left out.

The first graph has a break in the vertical axis, so comparing the bars may lead to incorrect conclusions. It looks as if attendance in 2002 was about three times as great as in 1998, but attendance only doubled during this time. The second graph is less likely to mislead.

EXAMPLE 2 Analyzing Misleading Graphs

Business The line graphs below display a company's profits and sales for each year from 1998 to 2002. What is misleading about each graph?

a.



b.

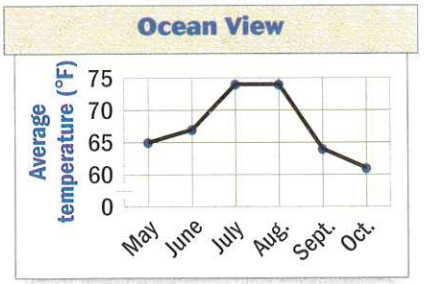
Solution

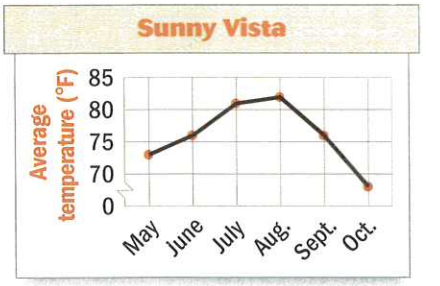
a. The range of values on the vertical scale is larger than needed. The graph suggests that profits have decreased only slightly, when they have decreased by a third.

b. Because the vertical axis starts at 300, it looks as if sales have risen rapidly. An unbroken axis would show that sales have risen slowly, at less than 2% per year.

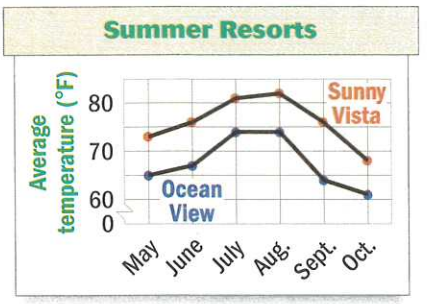
EXAMPLE 3 Misleading Comparisons

Compare the average monthly temperatures at two resorts.





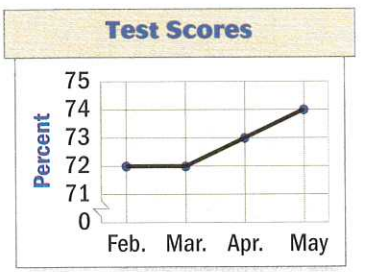
At a glance, the displays suggest that the temperatures are about the same, but the vertical axes are different. When data for both resorts are graphed together, it becomes clear that Sunny Vista has warmer temperatures.



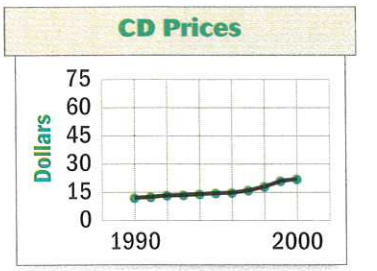
Exercises

Tell whether the data are represented clearly. If the graph is misleading, explain why. Then redraw the graph so that it is not misleading.

1.



2.



3. **Baseball** The table below shows a baseball pitcher's wins and losses per season. Make a graph that shows a positive performance. Then make another graph that shows a negative performance.

Year	1998	1999	2000	2001	2002
Wins	6	8	10	12	16
Losses	7	9	9	13	14

LESSONS 12.1 TO 12.3

Notebook Review

Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

stem-and-leaf plot, p. 597
box-and-whisker plot, p. 601
lower quartile, p. 601
upper quartile, p. 601
lower extreme, p. 601
upper extreme, p. 601
circle graph, p. 605
line graph, p. 606

Use Your Vocabulary

1. Copy and complete: The ? is the least value in a data set and the ? is the greatest value in the data set.

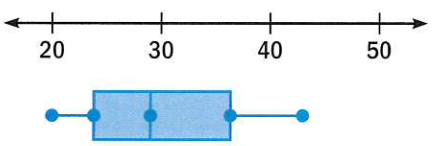
12.1–12.2 Can you order and display data?

Review

EXAMPLE Order the data set 24, 29, 35, 32, 22, 20, 43, 27, 41, 31, 26 in a stem-and-leaf plot. Then make a box-and-whisker plot of the data.

2	0 2 4 6 7 9
3	1 2 5
4	1 3

Key: 4 | 1 = 41

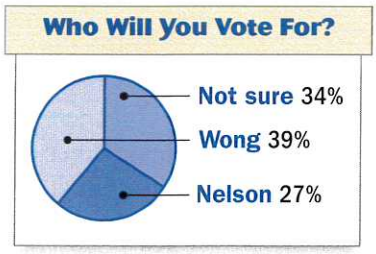


2. Make an ordered stem-and-leaf plot and a box-and-whisker plot of the wind speed data, given in miles per hour: 9, 15, 8, 19, 11, 18, 11, 24, 7, 14.

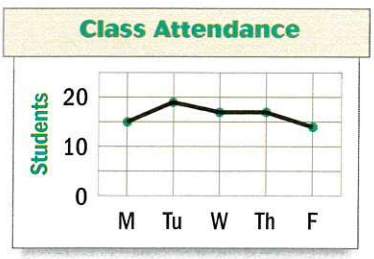
12.3 Can you use appropriate data displays?

Review

EXAMPLES



Use a circle graph to represent data as parts of a whole.



Use a line graph to display changes in a quantity over time.

3. Make a circle graph of survey results where 24% of the people surveyed said *no*, 47% said *yes*, and the rest said *not sure*.

4. The hourly temperatures, in degrees Celsius, starting at 1 P.M. were 33°C, 33°C, 34°C, 37°C, 37°C, and 36°C. Make a line graph of the data.

Stop and Think about Lessons 12.1–12.3

5. **Writing** Describe some real-world data that a double stem-and-leaf plot would help you analyze.

6. **Critical Thinking** Could you display the data from Exercise 3 in a line graph? Explain. What other type of display could be useful?

Review Quiz 1

1. **Attendance** Make an ordered double stem-and-leaf plot comparing the ballpark attendance data below for April and June. Then make a pair of box-and-whisker plots that compare the data. Use the displays to compare April attendance to June attendance.

   April: 1025, 1058, 1030, 997, 990, 1116, 1001, 995, 1122, 1099

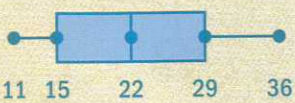
   June: 1056, 1125, 1151, 1048, 1123, 1097, 1042, 1164, 1125, 1131

2. **Nuts** Consumption of peanuts per person in the United States was as follows: 1995, 5.7 lb; 1996, 5.7 lb; 1997, 5.9 lb; 1998, 5.9 lb; 1999, 6.4 lb. Make a circle graph or a line graph of the data. Explain your choice.

BRAIN GAME

Safe Cracker

Use the box-and-whisker plot to find the missing leaves in the stem-and-leaf plot of the same group of eight numbers. Take the numbers that the missing leaves represent, and put them in order from least to greatest to find the combination that opens the safe.



1	1 4 ?
2	0 ? 7
3	1 ?

Key: 1 | 4 = 14

12.4 Problem Solving Strategies

- Guess, Check, and Revise
- Look for a Pattern
- Draw a Diagram
- Act It Out
- Solve a Simpler Problem
- Work Backward
- Make a Table

Solve a Simpler Problem

Problem There are 10 people in a room. They introduce themselves to each other by shaking every other person's hand one time. How many handshakes occur?

1 Read and Understand

Read the problem carefully.

You know that 10 people in a room each shake every other person's hand.

You want to find the total number of handshakes.

2 Make a Plan

Decide on a strategy to use.

By solving a series of simpler problems, you can identify a pattern in the number of handshakes. Then you can extend the pattern to find how many handshakes occur among 10 people.

3 Solve the Problem

Reread the problem and solve a simpler problem.

First, find the number of handshakes between 2 people, 3 people, and so on. You can do this by drawing a diagram. Make a table of your results.

Number of people	2	3	4	5
Number of handshakes	1	3	6	10
Pattern		+2	+3	+4

Extend the pattern to find the number of handshakes among 10 people.

1, 3, 6, 10, 15, 21, 28, 36, 45

+5 +6 +7 +8 +9

ANSWER So, 45 handshakes occur when 10 people shake hands.

4 Look Back

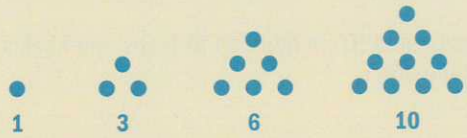
Double-check your calculations to be sure you didn't make any mistakes. You can also use the strategy *act it out* to check your answer.

Practice the Strategy

Use the strategy *solve a simpler problem.*

1. **Restaurants** A restaurant has 28 square tables that seat one person per side. You can join tables together. How many people can fit at two long tables made from all 28 tables? Will it make a difference if the tables are not divided equally?

2. **Triangular Numbers** The dots arranged in a triangle represent triangular numbers.



   Predict the number of dots in the eighth triangular number.

3. **Tournament** You are planning a small tournament for a chess club. If each of five members plays one game against each of the other members, how many games must you schedule?

4. **House Numbers** The houses on Stanford Street are numbered consecutively from 10 to 132. How many of each digit do you need to form all the house numbers?

5. **Odd Numbers** Find the sum of the first 50 odd whole numbers.

6. **Marching Band** A marching band is in a triangular formation. There is 1 band member in the front row. Each of the other rows contains 2 more band members than the row in front of it. There are 11 rows in all. How many band members are there?

7. **Palindromes** A palindrome is a number that reads the same backward and forward. For example, the number 1 and the number 414 are both palindromes. How many palindromes are there from 1 to 500?

Mixed Problem Solving

Use any strategy to solve the problem.

8. **Walking** Jay and Paul start walking in opposite directions. Jay walks 0.75 mile every 12 minutes, and Paul walks 2.5 miles every 30 minutes. How far apart are they after 1.5 hours?

9. **Games** What is the total number of squares on the checkerboard? Include squares of all sizes.

10. **Prisms** How does doubling the height of each base of a triangular prism affect its volume? How does halving the height of each base affect the volume?

11. **Calendars** What day of the week is the 3117th day after Thursday?

12. **Checking Account** The list below shows deposits and withdrawals this month. If Rodney has $57.68 in his checking account now, how much money did he have before Week 1?

    Week 1: He bought a shirt for $18 and a pair of pants for $26.

    Week 2: He bought a CD for $14 and lunch for $7.50.

    Week 3: He deposited $20.

LESSON 12.4

Counting Methods

BEFORE

You found theoretical and experimental probability.

NOW

You will use counting methods to count the number of choices.

WHY?

So you can count the outcomes of an election, as in Ex. 12.

Word Watch

tree diagram, p. 618

Activity You can count choices using an organized list.

You are choosing an outfit. You can choose a T-shirt (T), a button-down shirt (B), or a sweater (S) as a top and jeans (J) or khakis (K) for pants.

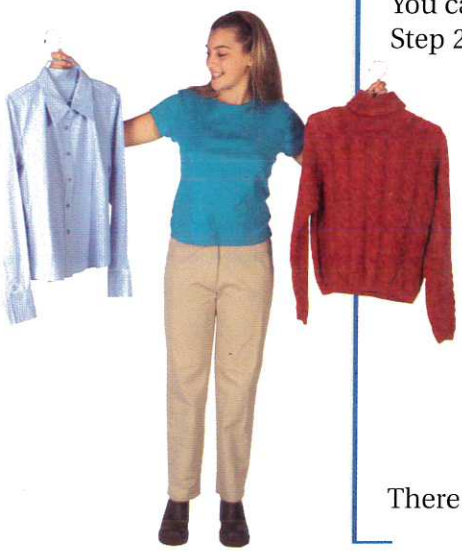
1. Use the letters to represent possible outfits. One possible outfit is JT, which means jeans and a T-shirt. Make a list of all possible outfits.

2. You decide to consider dress pants (D) in addition to jeans and khakis. How many outfits are possible now?

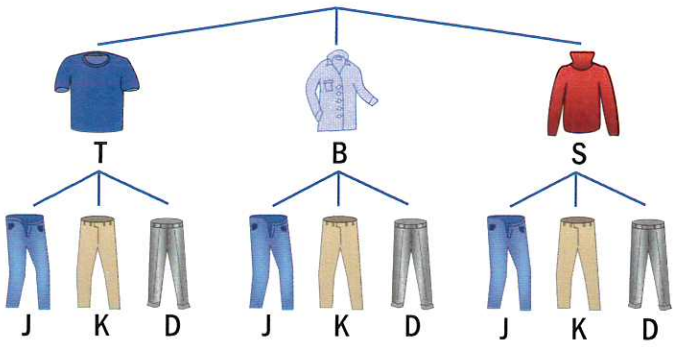
3. You also decide to include socks as part of the outfit. You can choose between red (R) and green (G). How many outfits are possible now?

In the activity, you made lists to count the number of choices. Another way to count the number of choices is to use a **tree diagram**.

EXAMPLE 1 Making a Tree Diagram



You can use a tree diagram to count the number of possible outfits in Step 2 of the activity above.



There are 9 different possible outfits.

Another way to count choices is to use the *counting principle.*

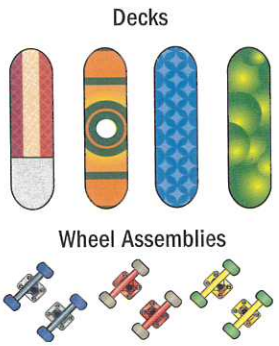
The Counting Principle

If one event can occur in m ways, and for each of these a second event can occur in n ways, then the number of ways that the two events can occur together is $m \cdot n$.

This principle can be extended to three or more events.

EXAMPLE 2 Using the Counting Principle

Skateboards To build a skateboard, you can choose one deck and one type of wheel assembly from those shown. To count the number of different skateboards you can build, use the counting principle.



$$\underset{\text{decks}}{4} \cdot \underset{\text{wheel assemblies}}{3} = 12 \qquad \text{Counting principle}$$

ANSWER You can build 12 different skateboards.

EXAMPLE 3 Using the Counting Principle

Passwords You are choosing a password that starts with 3 letters and then has 2 digits. How many different passwords are possible?

Solution

$$\underbrace{26 \cdot 26 \cdot 26}_{\text{letters}} \cdot \underbrace{10 \cdot 10}_{\text{digits}} = 1{,}757{,}600 \qquad \text{Counting principle}$$

ANSWER There are 1,757,600 different possible passwords.

Your turn now **Count the choices.**

1. Your soccer team's uniform choices include yellow and green shirts, white, black, and green shorts, and four colors of socks. Use a tree diagram to find how many different uniforms are possible.
2. In Example 3, suppose that the passwords may not start with an A or use the digit 0. How many different passwords are possible? Explain.

EXAMPLE 4 Finding a Probability

Number Cubes You and three friends each roll a number cube. What is the probability that you each roll the same number?

(1) List the *favorable* outcomes. There are 6:

1-1-1-1 2-2-2-2 3-3-3-3 4-4-4-4 5-5-5-5 6-6-6-6

(2) Use the counting principle to find the number of *possible* outcomes.

$$\underbrace{6 \cdot 6 \cdot 6 \cdot 6}_{\text{4 number cubes}} = 1296$$

(3) Then use the formula for finding probability.

$$\frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}} = \frac{6}{1296} = \frac{1}{216}$$

ANSWER The probability that you each roll the same number is $\frac{1}{216}$.

HELP with Review
For help with probability, see p. 354.

12.4 Exercises

More Practice, p. 738

Getting Ready to Practice

1. **Vocabulary** Copy and complete: If there are m ways that one event can occur and for each of those there are n ways that a second event can occur, then there are ? ways that the two events can occur together.

Make a tree diagram to find the number of choices that are possible. Then check your answer using the counting principle.

2. Choose apple, blueberry, lemon, or cherry pie with juice or tea.
3. Choose a small, medium, or large shirt in red, yellow, or blue.
4. Choose a car or truck with a tape player or a CD player.
5. Choose a hat or scarf in gray, white, or black.
6. **Weekend Plans** You would like to go to the movies, a play, or the zoo. You can invite your cousin or your best friend. You can go on Friday or Saturday. Make a tree diagram to list all of the possibilities. Then use the counting principle to check your answer.
7. **Movies** You and three friends all go to the movies on Friday night. You each pick a movie at random from the four choices. What is the probability that you all pick the same movie to see?

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	8–12, 14, 17
2	8–14, 18
3	8–14, 18
4	15–16

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In Exercises 8–11, make a tree diagram to find the number of choices that are possible. Then check your answer using the counting principle.

8. Choose a red, green, blue, or gray ball with a black or silver racquet.

9. Choose one of 6 DVDs and one of 4 CDs.

10. Choose one of 5 essays and one of 3 extra credit questions.

11. Choose a city tour or a park tour with passes to the art or science museum and a trip to the zoo, baseball game, movies, or mall.

12. Class Election The lists show the candidates for offices in a class election. Make a tree diagram to find the number of different ways a president, treasurer, and secretary can be chosen. Then use the counting principle to check your answer.

President	Treasurer	Secretary
☐ Amy	☐ Jessica	☐ Scott
☐ Hector	☐ Michael	☐ Nicole
	☐ Carson	☐ Thomas
		☐ Angela
		☐ Isabel

13. Banking You open a bank account and need to choose a password with 4 characters that can repeat. The password starts with 3 digits and then has 1 letter. How many different passwords are possible?

14. Writing There are 5 CDs and 4 books that you are interested in buying. Describe how to find the number of different pairs of 1 CD and 1 book that you can buy.

15. Coins Six people all flip a coin. What is the probability that they all get heads?

16. States You and Terry randomly choose the name of a state. What is the probability that you both choose a state whose name starts with a T? (Tennessee and Texas are the two states that start with the letter T.)

Extended Problem Solving **You are at a grocery store buying flavored water. You can choose lime, lemon, cherry, or orange. You can choose a 0.5-liter, 1-liter, or 2-liter bottle.**

17. Draw Make a tree diagram that shows all of the different bottles of flavored water that you can choose.

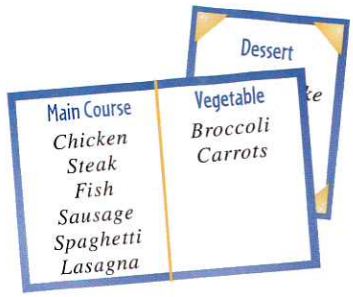
18. Multiply How many total different bottles can you choose from if 5 new flavors become available?

19. Challenge The sign shows the prices for each bottle size. What are the different total prices that you could be charged for three bottles? What is largest total quantity of water you can buy if you have only $2.60?

Water Sale
0.5 liter — $0.59
1 liter — $0.69
2 liter — $0.99

20. Solve a Simpler Problem If you have 5 cousins, and the probability that a cousin is a boy is 0.5, what is the probability that there is at least one boy and at least one girl among the cousins? Explain how you used a simpler problem to get your answer. (*Hint:* How many of the possible outcomes do *not* include both a boy and a girl?)

21. Critical Thinking A restaurant has 36 possible meal specials that you can choose. A meal has a main course, a vegetable, and a dessert. The restaurant has 6 different main courses and 2 different vegetables. How many different desserts does it have?



Braille **In Exercises 22 and 23, use the following information.**

Braille uses arrangements of raised dots to form symbols that represent letters, numbers, and punctuation marks. Braille is read by touching the symbols. Each symbol is a cell of 6 dots arranged in 3 rows of 2. In the cell, certain dots are raised to make a particular symbol.

22. How many different Braille symbols are possible? How many symbols are possible with no raised dots? With 6 raised dots?

23. Are the number of symbols possible with one raised dot the same as the number of symbols possible with 5 raised dots? Explain.

Mixed Review

Find the slope of the line passing through the points. *(Lesson 11.6)*

24. (5, 2), (−4, 2) **25.** (−2, 3), (4, 6) **26.** (3, 1), (7, −2)

27. The ages, in years, of youth group members are 12, 9, 8, 16, 12, 13, 8, 10, 11, and 17. Make a box-and-whisker plot of the data. *(Lesson 12.2)*

Test-Taking Practice

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28. Multiple Choice Your computer password has three digits. Which of the expressions would you use to find the total number of possible passwords?

A. 10 + 10 + 10 **B.** 10 • 10 • 10 **C.** 10 + 3 **D.** 10 • 3

29. Short Response You would like a sandwich, a side order, and a drink for lunch. You have a choice of a turkey, tuna, ham, or roast beef sandwich. You may have fruit, salad, or soup as a side order. You may choose juice or iced tea to drink. Make a tree diagram to show all the possible lunches that you can have. Write and evaluate an expression to find the total number of possible lunches.

LESSON 12.5

Permutations

BEFORE You used the counting principle to count possibilities.

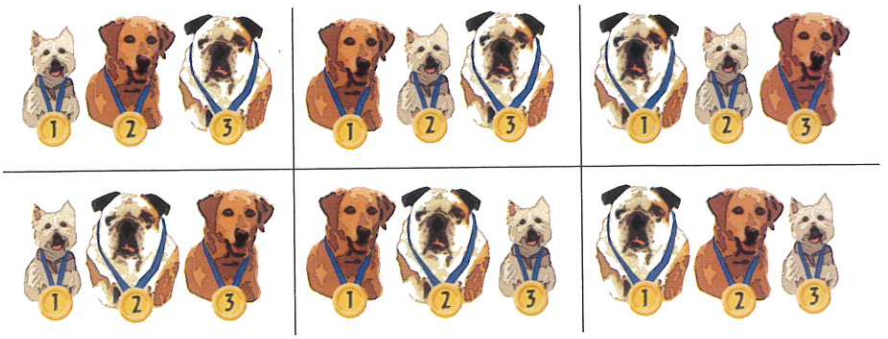
NOW You will use permutations to count possibilities.

WHY? So you can count the ways you can knit a hat, as in Ex. 20.

Word Watch

permutation, p. 623
factorial, p. 623

In some arrangements of groups, order is important. For example, the diagram shows the different ways that a group of three dogs could finish first, second, and third at a dog show.



Each arrangement lists the same dogs, but the orders are different. Arrangements such as these are called *permutations*. A **permutation** is an arrangement in which order is important. You can use the counting principle to count permutations.

EXAMPLE 1 Counting Permutations

Music You have five CDs. You can use the counting principle to count the number of permutations of 5 CDs. This is the number of different orders in which you can listen to the CDs.

Choices for 1st CD		Choices for 2nd CD		Choices for 3rd CD		Choices for 4th CD		Choices for 5th CD	
5	•	4	•	3	•	2	•	1	= 120

ANSWER You can listen to the CDs in 120 different orders.

HELP with Solving

You can use $n!$ to find the number of permutations of n objects.

Factorials In Example 1, you evaluated $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. You can write $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ as 5!, which is read "5 **factorial**."

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \qquad n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$$

The value of 0! is defined to be 1.

Your turn now **Evaluate the factorial.**

1. 3! **2.** 4! **3.** 6! **4.** 1!

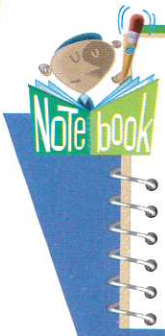
EXAMPLE 2 Counting Permutations

Band Competition Twelve marching bands are entered in a competition. You can use the counting principle to count how many ways first, second, and third places can be awarded.

Choices for 1st place		Choices for 2nd place		Choices for 3rd place		
12	•	11	•	10	= 1320	Counting principle

ANSWER There are 1320 ways to award the three places.

Permutation Notation Example 2 shows how to find the number of permutations of 12 objects taken 3 at a time. This is written ${}_{12}P_3$.



Permutations

Algebra The number of permutations of n objects taken r at a time can be written as ${}_nP_r$ and evaluated using $\frac{n!}{(n-r)!}$.

Numbers ${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 7 \cdot 6 \cdot 5$

EXAMPLE 3 Evaluating a Permutation

Poetry Two students are chosen from a group of 6 to read the first and second poems at the school's poetry reading. To find how many different ways the students can be chosen, find ${}_6P_2$.

$${}_6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!}$$ Use formula.

$$= \frac{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$ Divide out common factors.

$$= 30$$ Multiply.

ANSWER There are 30 ways the speakers can be chosen.

HELP with Solving

In Example 3, you can write 6! as 6 • 5 • 4! and cancel both 4 factorials.

$$\frac{6 \cdot 5 \cdot 4!}{4!} = 6 \cdot 5$$

Your turn now **Find the number of permutations.**

5. ${}_5P_3$ **6.** ${}_6P_6$ **7.** ${}_8P_7$ **8.** ${}_{100}P_2$

9. In Example 1 on page 623, you found the number of permutations of 5 CDs taken how many at a time? Explain.

12.5 Exercises

More Practice, p. 738

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Getting Ready to Practice

1. **Vocabulary** Copy and complete: The number of permutations of 15 objects taken 7 at a time can be written as ?.

Evaluate.

2. $2!$
3. $0!$
4. $7!$
5. $9!$
6. ${}_4P_2$
7. ${}_9P_6$
8. ${}_{10}P_7$
9. ${}_5P_5$
10. You have three things to do after you wake up tomorrow. You need to take a shower, eat breakfast, and call your friend. Find the number of permutations and list them.

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	11
2	20-22, 24
3	12-22, 24

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11. **Movies** You rent four movies. In how many different orders can you watch the movies?

Find the number of permutations.

12. ${}_3P_1$
13. ${}_5P_2$
14. ${}_3P_2$
15. ${}_9P_3$
16. ${}_{12}P_6$
17. ${}_7P_4$
18. ${}_{15}P_5$
19. ${}_{20}P_3$
20. **Knitting** You are knitting a hat, and you want it to have 3 different colored stripes. You have 6 different colors of yarn. How many different hats could you knit?
21. **Softball** Your softball team has 15 players. Find the number of different ways that the first, second, third, fourth, and fifth batters can be chosen.
22. **After School** You are given a list of 10 activities you can do after school. You are asked to pick your first, second, third, and fourth choices. How many different permutations are possible?
23. **Critical Thinking** Your friend says that $11! = 11 \cdot 10!$. Is your friend correct? Explain.
24. **Gardening** You are planning a garden with 3 rows. Each row will have one type of flower, and none of the rows will be the same. You can choose from the flowers below. Find the number of permutations.

Day Lily, Poppy, Gladiolus, Daffodil, Rose, Sunflower, Tulip

Extended Problem Solving **In Exercises 25 and 26, your team is 1 of 15 in a cheerleading competition.**

25. If trophies are awarded for first, second, third, and fourth places, in how many different ways can the trophies be awarded?

26. **Compare and Contrast** Suppose the four teams that perform best are all given *excellence* medals instead of first, second, third, and fourth place trophies. In this case, are there more or fewer ways to give the awards than in Exercise 25? Explain.

27. **Challenge** You choose a 6-letter password for your e-mail. Write an expression to represent the number of different passwords you could choose. Explain how this expression changes if you can use each letter only once.

Mixed Review

28. Find the surface area of a cone that has a slant height of 24 inches and a diameter of 10 inches. Round to the nearest tenth. *(Lesson 10.5)*

29. The table shows the results of a survey that asked students to choose their favorite color. Make a circle graph of the data. *(Lesson 12.3)*

Red	25%
Green	12%
Blue	40%
Other	23%

30. You are making a cake for your younger sister's birthday. You can make vanilla or chocolate cake. You can have white, yellow, or blue frosting. You can make a balloon design or a flower design for the top of the cake. Make a tree diagram to show all of the possible cakes that you could make. *(Lesson 12.4)*

Test-Taking Practice

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31. **Multiple Choice** You go to the cafeteria with five friends. In how many different orders can you and your friends get into the lunch line?

A. 6 **B.** 30 **C.** 120 **D.** 720

32. **Short Response** When you come back from vacation, you want to call Ed, Sue, Ty, and Nestor. You have time to make only two calls. Find the number of different orders in which you can call two friends. Then make a list to show all the different orders.

LESSON 12.6

Combinations

BEFORE You used permutations to count possibilities.

NOW You will use combinations to count possibilities.

WHY? So you can count the ways a team can choose captains, as in Ex. 22.

Word Watch

combination, p. 627

In Lesson 12.5, you studied permutations, which are arrangements in which order is important.

A **combination** is a group of items whose order is *not* important. For example, suppose you go to lunch with a friend. You choose milk, soup, and a salad. Your friend chooses soup, a salad, and milk. The order in which the items are chosen does not matter. You both have same meal.

EXAMPLE 1 Listing Combinations

County Fair You have 4 tickets to the county fair and can take 3 of your friends. You can choose from Abby (A), Brian (B), Chloe (C), and David (D). How many different choices of groups of friends do you have?

Solution

List all possible arrangements of three friends. Then cross out any duplicate groupings that represent the same group of friends.

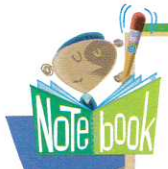
(ABC)	~~ACB~~	~~BAC~~	~~BCA~~	~~CAB~~	~~CBA~~	← ABC, ACB, BAC, BCA, CAB, and CBA all represent the same group.
(ABD)	~~ADB~~	~~BAD~~	~~BDA~~	~~DAB~~	~~DBA~~	
(ACD)	~~ADC~~	~~CAD~~	~~CDA~~	~~DAC~~	~~DCA~~	
(BCD)	~~BDC~~	~~CBD~~	~~CDB~~	~~DBC~~	~~DCB~~	

ANSWER You have 4 different choices of groups to take to the fair.

Your turn now

1. In Example 1, the complete list shows the number of *permutations* of 4 items chosen 3 at a time. How many items would be in the complete list if you had to choose from 8 friends?

Combination Notation In Example 1, after you cross out the duplicate groupings, you are left with the number of combinations of 4 items chosen 3 at a time. Using notation, this is written ${}_4C_3$.



Combination Notation

Words To find the number of combinations of n objects taken r at a time, divide the number of permutations of n objects taken r at a time by $r!$.

Numbers ${}_9C_4 = \frac{{}_9P_4}{4!}$ **Algebra** ${}_nC_r = \frac{{}_nP_r}{r!}$

EXAMPLE 2 Evaluating Combinations

Find the number of combinations.

a. ${}_8C_3$ **b.** ${}_9C_7$

Solution

a. ${}_8C_3 = \frac{{}_8P_3}{3!}$ Combination formula

$= \frac{8 \cdot 7 \cdot 6}{3!}$ ${}_8P_3 = \frac{8!}{(8-3)!} = 8 \cdot 7 \cdot 6$

$= \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3} \cdot \cancel{2} \cdot 1}$ Expand. $3! = 3 \cdot 2 \cdot 1$. Divide out common factors.

$= 56$ Simplify.

b. ${}_9C_7 = \frac{{}_9P_7}{7!}$ Combination formula

$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{7!}$ ${}_9P_7 = \frac{9!}{(9-7)!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$

$= \frac{9 \cdot \overset{4}{\cancel{8}} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \underset{1}{\cancel{2}} \cdot 1}$ Expand 7!. Divide out common factors.

$= 36$ Simplify.

Your turn now **Find the number of combinations.**

2. ${}_8C_8$ **3.** ${}_8C_7$ **4.** ${}_7C_2$ **5.** ${}_6C_1$

EXAMPLE 3 Permutations and Combinations

Tell whether the possibilities can be counted using a *permutation* or *combination*. Then write an expression for the number of possibilities.

a. Swimming There are 8 swimmers in the 400 meter freestyle race. In how many ways can the swimmers finish first, second, and third?

b. Track Your track team has 6 runners available for the 4-person relay event. How many different 4-person teams can be chosen?

Solution

a. Because the swimmers can finish first, second, or third, order is important. So the possibilities can be counted by evaluating ${}_8P_3$.

b. Order is not important in choosing the team members, so the possibilities can be counted by evaluating ${}_6C_4$.

HELP with Notetaking

Mutate means to change. For *permutations*, you count changes in the order of items. For *combinations*, objects *combined* in any order represent the same group. You may wish to copy this hint into your notebook.

12.6 Exercises

More Practice, p. 738

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Getting Ready to Practice

1. Vocabulary Copy and complete: The expression ${}_9C_5$ represents the number of combinations of ? objects taken ? at a time.

2. Find the Error You choose 3 art projects from the following list: clay, plaster, wood, wire, drawing, painting. Describe and correct the error below in finding the number of possible combinations of 3 projects.

✗ 1st Choice 2nd Choice 3rd Choice

$6 \cdot 5 \cdot 4 = 120$ ways to choose

Find the number of combinations.

3. ${}_4C_1$ **4.** ${}_4C_4$ **5.** ${}_7C_6$ **6.** ${}_5C_2$

Tell whether the possibilities should be counted using a *permutation* or *combination*. Then find the answer.

7. Party You are buying balloons for a party. The store has four different colors, and you would like to choose two different colors. How many different pairs of balloon colors can be chosen?

8. Homework You must do homework in math, history, science, and geography. In how many different orders can you do your homework?

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	9
2	10–21, 22–25
3	26–29

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9. Essays For a test, you can choose any 2 essay questions to answer from the 5 questions asked. Make a list and cross out the duplicate choices to show how many different pairs of essay questions you could answer.

Find the number of combinations.

10. ${}_6C_5$ **11.** ${}_6C_6$ **12.** ${}_{11}C_9$ **13.** ${}_8C_4$

14. ${}_8C_6$ **15.** ${}_8C_1$ **16.** ${}_{10}C_8$ **17.** ${}_9C_5$

18. ${}_{11}C_3$ **19.** ${}_{13}C_{11}$ **20.** ${}_9C_2$ **21.** ${}_{100}C_{99}$

22. Hockey Your hockey team is choosing 2 team captains from its 18 members. Find the number of combinations that are possible.

23. Debating A debate team has 5 members. Your debating club has 12 students. How many different teams can be chosen?

24. Gardening You want to grow 4 different vegetables. You can choose from 9 types of seed. Find the number of combinations of 4 vegetables.

25. Writing Is it possible to evaluate a combination such as ${}_3C_4$, ${}_2C_6$, or ${}_1C_{10}$? Explain why or why not.

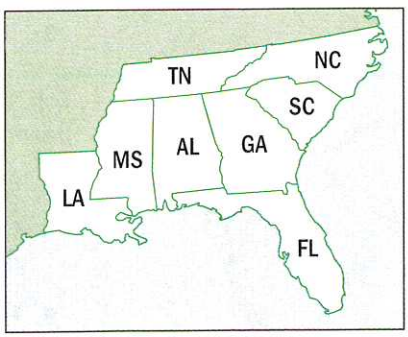
In Exercises 26–29, tell whether the possibilities should be counted using a *permutation* or *combination*. Then find the number of possibilities.

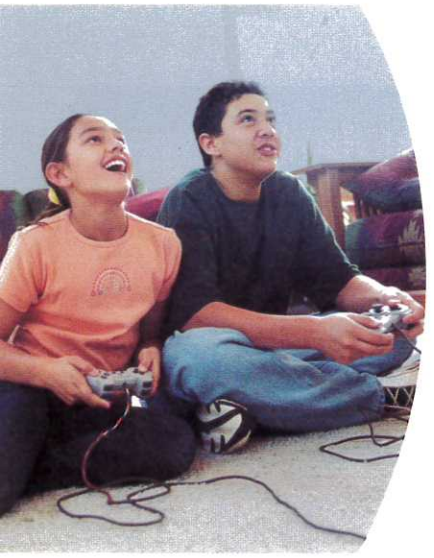
26. Music You want to know the number of ways you can play your four favorite songs.

27. Shopping You are shopping for a trip and want to buy three sweaters from among a red sweater, a blue sweater, a plaid sweater, a striped sweater, and a turtleneck sweater. How many sets of three sweaters can you choose?

28. School Colors Your class is voting for the new school colors. You are asked to choose 2 colors from a list of 8 colors. How many possibilities are there?

29. Geography You are coloring the map shown at the right. You want each state to be a different color, and you have 10 possible colors. In how many ways can you color the map? In how many ways can you choose 8 colors?





30. Look for a Pattern Copy the table. Complete the table by finding the number of combinations. Then describe the pattern.

${}_7C_0$	${}_7C_1$	${}_7C_2$	${}_7C_3$	${}_7C_4$	${}_7C_5$	${}_7C_6$	${}_7C_7$
?	?	?	?	?	?	?	?

Video Games In Exercises 31 and 32, your friend has a collection of 20 video games.

31. You want to borrow four games from your friend. How many different groups of four games can you choose?

32. If you are already sure about two of the game choices, how many different groups of four games can you choose? Explain.

33. Challenge What is the value of ${}_nC_r$ when $r = n$? What is the value of ${}_nC_r$ when $n - r = 1$? Explain.

Mixed Review

Choose a Strategy Use a strategy from the list to solve the following problem. Explain your choice of strategy.

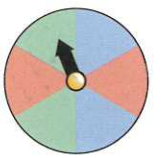
Problem Solving Strategies
- Act It Out
- Solve a Simpler Problem
- Draw a Diagram
- Make a Table

34. A rectangular room is 20 feet by 12 feet, with walls 7.5 feet high. A decorator charges \$.40 per square foot to use paint and \$.70 per square foot for wallpaper. Find the cost to decorate the room with each material.

Find the number of permutations. *(Lesson 12.5)*

35. ${}_{10}P_5$ **36.** ${}_{11}P_4$ **37.** ${}_{18}P_3$ **38.** ${}_{21}P_2$

39. Basic Skills You spin the spinner at the right. What is the probability that the spinner lands on blue?



Test-Taking Practice

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40. Multiple Choice You are at a fair with four friends. All of you want to ride the roller coaster, but only three people can fit in the first car. How many different groups of three can you and your friends make?

A. 60 **B.** 24 **C.** 10 **D.** 4

41. Short Response You have 6 different sweatshirts, and you want to donate some to a charity. Draw a diagram or write an expression so that you can find the number of ways you can donate 2 sweatshirts. Then find the number of ways you can donate 3 sweatshirts.

LESSON 12.7

Probability and Odds

BEFORE

You found the probability of events.

Now

You will find the odds in favor of events.

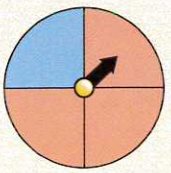
WHY?

So you can find the odds of a goalie's save, as in Ex. 18.

Word Watch

complementary events, p. 632
unfavorable outcome, p. 633
odds, p. 633

Activity **You can use a spinner to explore probability.**



1. What is the probability of the spinner landing on blue? What is the probability of the spinner landing on red?
2. Is it more likely that the spinner will land on blue than red?
3. Find each ratio. Use the ratios to compare the chances of landing on red and on blue.

   a. $\frac{\text{number of red sections}}{\text{number of blue sections}}$ b. $\frac{\text{number of blue sections}}{\text{number of red sections}}$

4. Explain how to find the probability of the spinner *not* landing on red.

In the activity, the spinner will land on either red or blue. Two events are **complementary** when one event or the other (but not both) must occur. The sum of the probabilities of complementary events is always 1.

When Events A and B are complementary, $P(\text{Event A}) = 1 - P(\text{Event B})$.

HELP with Solving

The probability that Event A occurs and the probability that Event A does *not* occur have a sum of 1 because they are complementary events.

For help with probability, see p. 354.

EXAMPLE 1 Finding Probabilities

Gifts You and seven friends contribute money for a gift. Everyone's name is put in a hat. The person whose name is chosen picks the gift.

a. What is the probability that your name is randomly chosen?

b. What is the probability that your name is randomly *not* chosen?

Solution

a. $P(\text{your name is chosen}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}} = \frac{1}{8}$

b. $P(\text{your name is not chosen}) = 1 - P(\text{your name is chosen})$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Your turn now **You are given the probability that an event will occur. Find the probability that the event will not occur.**

1. $P(\text{A}) = \frac{3}{4}$ **2.** $P(\text{A}) = 0.45$ **3.** $P(\text{A}) = 32\%$ **4.** $P(\text{A}) = \frac{7}{10}$

5. The 11 letters in the word MISSISSIPPI are each written on pieces of paper and put in a bag. What is the probability of randomly drawing an S from the bag? What is the probability of randomly *not* drawing an S?

Odds Once you specify the event for which you are finding the probability, outcomes for that event are called *favorable outcomes*. The other outcomes are **unfavorable outcomes**.

When all outcomes are equally likely, the **odds** in favor of an event are equal to the ratio of favorable outcomes to unfavorable outcomes.

$$\text{Odds} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}}$$

EXAMPLE 2 Finding Odds

You do a survey asking your class to rank three ice cream flavors. Results for vanilla are shown at the right. What are the odds in favor of a randomly chosen student from your class ranking vanilla first?

Solution

Vanilla was ranked first by 6 students, so there are 6 favorable outcomes.

It was ranked second by 12 students, and ranked third by 9 students, so there are $12 + 9 = 21$ unfavorable outcomes.

$$\text{Odds} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}}$$

$$= \frac{6}{21}$$

$$= \frac{2}{7}$$

ANSWER The odds in favor of a randomly chosen student ranking vanilla first are 2 to 7.

HELP with Reading

Odds are always read as a ratio. For example, $\frac{5}{2}$ is read "five to two," not "five halves."

Your turn now **You choose a card at random from a set of cards numbered 1 to 24. Find the odds in favor of the event.**

6. You choose a 10. **7.** You choose an odd number greater than 7.

Probability and Odds If you know the probability of an event, you can use the following formula to find the odds in favor of that event.

$$\text{Odds} = \frac{\text{Probability event will occur}}{\text{Probability event will not occur}} = \frac{\text{Probability event will occur}}{1 - \text{Probability event will occur}}$$

EXAMPLE 3 Finding Odds Using Probability

Basketball Sean makes 65% of his free throws. What are the odds in favor of Sean making a free throw?

Solution

$$\text{Odds} = \frac{0.65}{1 - 0.65}$$ Write percents as decimals.

$$= \frac{0.65}{0.35}$$ Subtract.

$$= \frac{65}{35}$$ Multiply numerator and denominator by 100.

$$= \frac{13}{7}$$ Simplify.

ANSWER Sean's odds in favor of making a free throw are 13 to 7.

12.7 Exercises

More Practice, p. 738

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Getting Ready to Practice

1. **Vocabulary** Copy and complete: Find the ratio of the number of favorable outcomes to the number of unfavorable outcomes to find the ? of an event.

You are given the probability that an event will occur. Find the probability that the event will not occur.

2. $P(\text{A}) = 84\%$ 3. $P(\text{A}) = \frac{2}{5}$ 4. $P(\text{A}) = 0.37$ 5. $P(\text{A}) = \frac{9}{10}$

You randomly draw a letter tile from a bag. The 8 letters in the word GEOMETRY are in the bag. Find the odds in favor of the event.

6. You choose a G. 7. You choose an E. 8. You choose an S.

9. **Weather** The weather forecast says that there is a 30 percent probability of rain. What are the odds in favor of rain?

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	10–13, 19–20
2	10–20
3	18, 20

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Find the probability of randomly choosing a red marble from the bag of marbles described. Then find the odds in favor of randomly choosing a blue marble.

10. 3 red and 7 blue marbles

11. 4 red and 9 blue marbles

12. 6 red and 5 blue marbles

13. 6 red, 5 blue, and 3 green marbles

Find the odds in favor of the event described when rolling a number cube.

14. Roll a 3.

15. Roll a number less than 6.

16. Roll a number greater than 2.

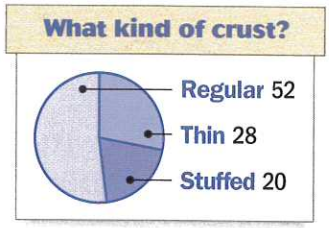
17. Roll an odd number less than 5.

18. Hockey A hockey goalie has a save percentage of 93%. What are the odds that he makes a save?

19. Compare Sam finds the probability of Event A is 0.2. Jan finds the odds in favor of Event A are 1 to 4. Can they both be right? Explain.

20. Socks In your sock drawer, you have 20 socks. You have 2 pairs of patterned socks, 4 pairs of gym socks, 3 pairs of striped socks, and 1 pair of black socks. What is the probability that you randomly pull a gym sock from the drawer? What are the odds?

Pizza In Exercises 21–23, use the circle graph. It shows the number of people ordering pizza that order each type of crust.



21. What is the probability that a randomly chosen pizza order is for thin crust?

22. What are the odds in favor of a randomly chosen pizza order being for regular crust?

23. What are the odds in favor of a randomly chosen pizza order *not* being for stuffed crust?

Odds Against In Exercises 24 and 25, use the following information. In this lesson, you learned how to find the *odds in favor* of an event. You can also find the *odds against* an event.

$$\text{Odds against} = \frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}}$$

24. You choose a chip from a bag of 6 blue, 3 red, and 5 green chips. Find the odds in favor of choosing a green chip. Then find the odds against choosing a green chip.

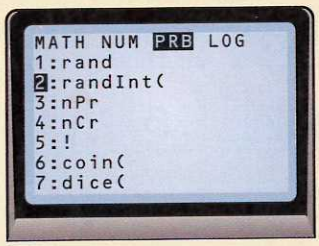
25. Challenge You hear a friend claim that "the odds that you get hit by lightning are a million to one." Is your friend talking about *odds in favor* or *odds against*? Explain your reasoning.

Explore 2 Use technology to simulate a real-world situation.

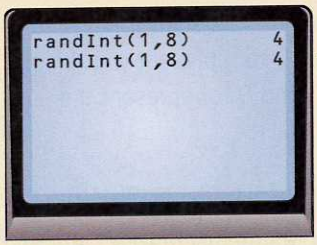
Use your calculator's random integer function to simulate randomly choosing 1 student from a group of 8. Find the experimental probability that any of the students is randomly chosen twice in a row.

1. Use 1, 2, 3, 4, 5, 6, 7, and 8 to represent the students.

2. Clear your calculator screen. Press MATH and choose the PRB menu. Select *randInt(*, the random integer function.



3. Enter **1** , **8** ENTER) to select an integer at random from 1 to 8. Press ENTER again to simulate choosing a student again. Record whether your results represent a match.



Notice that getting a 4 and a 4 represents choosing the same student twice in a row.

4. Do the simulation a total of 10 times. Record your results. What is the experimental probability that any of the students is randomly chosen twice in a row?

Your turn now Design a simulation of the situation.

2. You randomly choose to go to the library, from the choices mall, library, or bowling lanes.

3. You and your friend Chris are randomly chosen from a group of 10 team members to be co-captains.

Stop and Think

4. **Critical Thinking** Refer to Exercise 1 on page 637. Which results do you think are more likely to be close to the theoretical probability that you and your friend are chosen, your results or the class results? Explain.

5. **Writing** Explain how the simulation in Explore 2 above would be different if you wanted to find the probability that you are randomly chosen twice in a row from a group of eight people.

LESSON 12.8

Independent and Dependent Events

BEFORE You found the probability of an event.

NOW You will study independent and dependent events.

WHY? So you can find the probability of winning a free snack, as in Ex. 13.

Word Watch

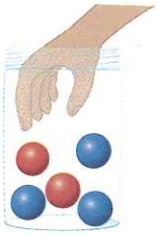
independent events, p. 639
dependent events, p. 639

Two events are **independent events** if the occurrence of one event does *not* affect the probability that the other event will occur. Two events are **dependent events** if the occurrence of one event *does* affect the probability that the other event will occur.

Suppose you randomly choose two gumballs one at a time from the jar below. The probability of choosing two red gumballs with replacement is different than the probability without replacement.

Independent Events

First Event



$P(\text{red}) = \frac{2}{5}$

Second Event

$P(\text{red}) = \frac{2}{5}$

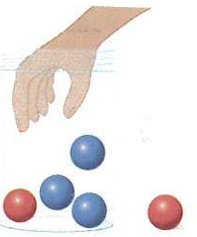
If you replace the gumball, the probability of choosing a red gumball is the same for each choice.

Dependent Events

First Event

$P(\text{red}) = \frac{2}{5}$

Second Event



$P(\text{red}) = \frac{1}{4}$

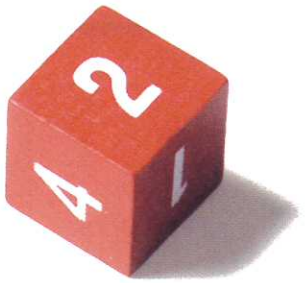
If you don't replace the gumball after choosing, the probability changes.

EXAMPLE 1 Independent and Dependent Events

Tell whether the events are *independent* or *dependent*.

a. You roll a number cube. Then you roll the number cube again.

b. You randomly draw a number from a bag. Then you randomly draw a second number without putting the first number back.

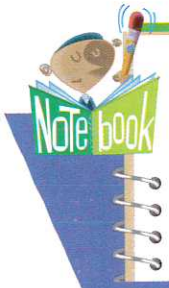


Solution

a. The result of the first roll does not affect the result of the second roll, so the events are independent.

b. There is one fewer number in the bag for the second draw, so the events are dependent.

Multiple Events To find the probability that Event A *and* Event B happen, you multiply probabilities. Because the occurrence of an event may affect the probability of another event, you should determine whether the events are independent or dependent before multiplying.



Probability of Independent Events

For two independent events, the probability that both occur is the product of the probabilities of the events.

$P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$ **Events A and B are independent.**

EXAMPLE 2 Probability of Independent Events

School Fair Your class is raising money by operating a ball toss game. You estimate that about 1 out of every 25 balls tossed results in a win. What is the probability that someone will win on two tosses in a row?

Solution

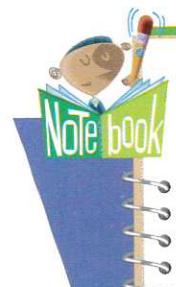
The tosses are independent events, because the outcome of a toss does not affect the probability of the next toss resulting in a win.

So the probability of each event is $\frac{1}{25}$.

$$P(\text{win and win}) = P(\text{win}) \cdot P(\text{win}) = \frac{1}{25} \cdot \frac{1}{25} = \frac{1}{625}$$

ANSWER The probability of two winning tosses in a row is $\frac{1}{625}$.

Dependent Events If A and B are dependent events, the probability that B occurs given that A also occurs is not the same as the probability of B. So, you should use $P(\text{B given A})$ instead of $P(\text{B})$ to represent the probability that B occurs given that A also occurs.

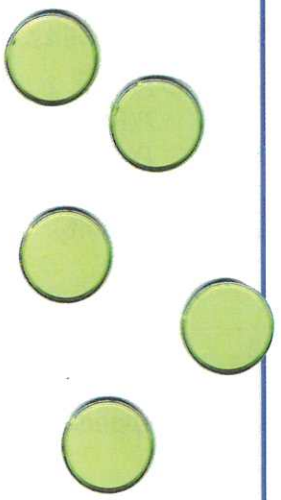


Probability of Dependent Events

For two dependent events, the probability that both events occur is the product of the probability of the first event and the probability of the second event given that the first event also occurs.

$P(\text{A and B}) = P(\text{A}) \cdot P(\text{B given A})$ **A and B are dependent.**

EXAMPLE 3 Finding Probability of Dependent Events



Bingo You are playing the bingo card shown. The caller has 50 numbers left to call. What is the probability that you will get bingo on the next 2 numbers called?

Solution

You need B7 and N44 for bingo. Find the probability of success when each of the next 2 numbers is drawn. Then multiply.

$P(\text{B7 or N44}) = \frac{2}{50} = \frac{1}{25}$ — There are 50 numbers left to call.

$P(\text{remaining number}) = \frac{1}{49}$ — There are 49 numbers left to call.

$P(\text{both numbers}) = \frac{1}{25} \cdot \frac{1}{49} = \frac{1}{1225}$ — Multiply the probabilities.

ANSWER The probability is $\frac{1}{1225}$, or about 0.0008.

Your turn now **Find the probability.**

1. You toss a coin twice. Find the probability of getting two heads.
2. Find the probability for Example 3 if there are 36 numbers left to call.

12.8 Exercises

More Practice, p. 738

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Getting Ready to Practice

1. **Vocabulary** Copy and complete: When the occurrence of an event does not affect the probability of the next event, the events are ?.

Tell whether the events are *independent* or *dependent*. Then find the probability.

2. You randomly choose a green marble from a jar of 8 green and 3 blue marbles. You replace the marble and randomly choose another green marble.
3. Your teacher randomly chooses you to give a report. She then randomly chooses Pam from the 22 remaining students.

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	10–14
2	4–6, 10–14
3	7–14

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Events A and B are independent. Find the missing probability.

4. $P(A) = 0.4$
   $P(B) = 0.6$
   $P(A \text{ and } B) = \underline{?}$

5. $P(A) = 0.9$
   $P(B) = \underline{?}$
   $P(A \text{ and } B) = 0.09$

6. $P(A) = \underline{?}$
   $P(B) = 0.6$
   $P(A \text{ and } B) = 0.12$

Events A and B are dependent. Find the missing probability.

7. $P(A) = 0.75$
   $P(B \text{ given } A) = 0.5$
   $P(A \text{ and } B) = \underline{?}$

8. $P(A) = 0.8$
   $P(B \text{ given } A) = \underline{?}$
   $P(A \text{ and } B) = 0.32$

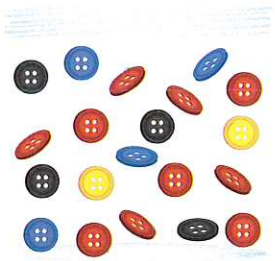
9. $P(A) = \underline{?}$
   $P(B \text{ given } A) = 0.3$
   $P(A \text{ and } B) = 0.039$

In Exercises 10–12, tell whether the events are *independent* or *dependent*. Then find the probability.

10. **Banquet** At a banquet, you can order a main course of a chef's salad, salmon and potatoes, ham and beans, or steak and rice. You can drink water, juice, milk, coffee, or iced tea. If all choices are equally likely, what is the probability that a randomly chosen person orders a chef's salad and juice?

11. **Cookies** You have a jar filled with 5 oatmeal cookies, 6 sugar cookies, 8 frosted cookies, and 9 chocolate cookies. You randomly choose a cookie, keep it, and then choose another. What is the probability that you pick a frosted cookie and then a chocolate cookie?

12. **Buttons** You draw a button at random from the jar at the right. Without replacing the first button, you draw another. What is the probability that you draw a red button and then a yellow button?



13. **Lucky Plate** Each day, the person who gets the lucky plate wins a free snack from the school cafeteria. The cafeteria sells 127 lunches on Wednesday and 134 lunches on Thursday. What is the probability that you win on both days if you buy lunch both days?

14. **Critical Thinking** What is the probability that when you toss a coin you get heads 7 times in a row? If you have already gotten heads 6 times in a row, what is the probability that you will get heads on the next toss?

15. **Challenge** A brochure says, "If you invested money with us 5 years ago, that money grew by an average of 20%." The brochure also says, "Past performance is no guarantee of future results." Which statement leads you to think that investing money today is independent of past events? Which statement suggests the opposite? Explain.

16. Elections A town's election for mayor drew 75% of the town's 800 eligible voters. What is the probability that two different randomly selected people both voted in the election?

Mixed Review

17. The table shows the amount of time, in minutes, that Cindy ran on the treadmill each day. Make a line graph of the data. Predict how long Cindy will run on Saturday. Explain your reasoning. *(Lesson 12.3)*

Monday	20
Tuesday	25
Wednesday	25
Thursday	35
Friday	41

Basic Skills **Evaluate the expression.**

18. $5 \cdot (11 - 4)$ **19.** $3 + 9 \cdot 6$ **20.** $35 - 21 \div 3$ **21.** $\frac{35}{9 - 2}$

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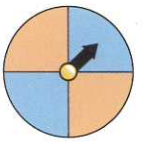
22. Multiple Choice Suppose you spin the spinner at the right twice. What is the probability of landing on a blue region both times?



A. $\frac{1}{16}$ **B.** $\frac{1}{8}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{2}$

23. Short Response Your teacher is giving away two prizes by random drawing. She puts 25 students' names into a hat. She draws the first name. Then she chooses a second name without replacing the first name. What is the probability that your name will be chosen first? Does the probability that your name will be chosen second depend on the first outcome? Explain why or why not.

BRAIN GAME

Lucky Numbers

Two balls will be randomly chosen without replacement from the globe shown.

Bo wins if the first ball is blue, and the next ball is a 3 or a 4.

Sherry wins if the first ball is an even number and the next ball is green.

Eva wins if both balls are red.

Who has the best chance of winning?

CHAPTER 12 Special Topic

Samples

GOAL Identify biased samples and surveys.

Word Watch

population, p. 644
sample, p. 644
random sample, p. 644
biased sample, p. 644

One way to collect data about a group is by doing a survey. A **population** is the entire group of people or objects that you want information about. When it is difficult to survey an entire population, a **sample**, or a part of the entire group, is surveyed.

In a **random sample**, each person or object has an equally likely chance of being selected. A non-random sample can result in a **biased sample** that is not representative of the population.

EXAMPLE 1 Identifying Potentially Biased Samples

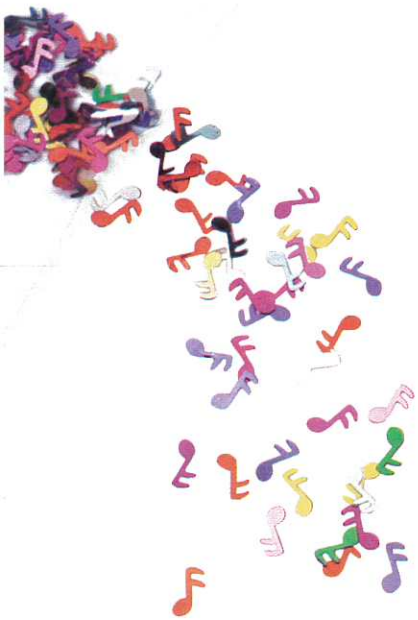
Costume Dance The student council wants students to help decide on a theme for a costume dance. Students can choose one of the council's three ideas from the options listed at the right.

Surveying all of the students will take too long, so a sample will be surveyed. Tell whether the survey method could result in a biased sample. Explain.

a. Survey members of the movie club.

b. Survey students as they enter the school.

c. Survey students on the football team.



Solution

a. This method could result in a biased sample because this group is more likely to favor the movie theme.

b. This method is not likely to result in a biased sample because a wide range of students will be surveyed.

c. This method could result in a biased sample because the football players are more likely to favor sports and games.

Survey Questions When you do a survey, you need to phrase the questions so that the responses of the people surveyed accurately reflect their opinions or actions. If not, claims based on the survey results may be biased.

EXAMPLE 2 Identifying Potentially Biased Questions

Tell whether the question could produce biased results. Explain.

a. Do you support the unfair policy of requiring students to do a time-consuming community project? YES ☐ NO ☐

b. Do you like our new apple-nut yogurt flavor, now on sale in stores everywhere? YES ☐ NO ☐

Solution

a. This question suggests that the policy is unfair and that the project is time-consuming. It encourages a response of *no*. So, the question could lead to biased results.

b. The question assumes that the person responding has tried the new yogurt flavor. Those who have not tried the new flavor may not give an accurate opinion. So, the question could lead to biased results.

Exercises

Stadiums In Exercises 1–3, a city wants to know whether residents favor using public funds to pay for a new baseball stadium. Tell whether the method could result in a biased sample. Explain.

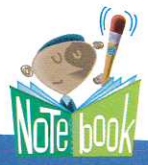
1. Ask people that call in to a sports radio talk show.
2. Ask every tenth person listed in the phone book.
3. Ask every fifth person who enters the sporting goods store in town.
4. **Food** A restaurant wants to know what kinds of food to add to its menu to attract new customers. Describe a sampling method that the restaurant can use that is not likely to result in a biased sample.

Tell whether the survey question could produce biased results. Explain your reasoning.

5. Would you rather relax at home while reading a book, or go to a noisy, crowded mall?
6. Allowing messy, dangerous dogs into the park will cause safety and health problems. Will you vote to allow dogs into the park?
7. How often do you buy lunch in the school cafeteria?
8. Do you agree with this store's policy for returning purchases?

LESSONS 12.4 TO 12.8

Notebook Review



Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

tree diagram, p. 618
permutation, p. 623
factorial, p. 623
combination, p. 627
complementary events, p. 632
unfavorable outcome, p. 633
odds, p. 633
independent events, p. 639
dependent events, p. 639

Use Your Vocabulary

1. **Vocabulary** Copy and complete: A(n) ? is an arrangement in which order is important.

12.4 Can you use the counting principle?

Review

EXAMPLE You need to choose one of 3 birdhouse designs and one of 6 possible colors. How many different birdhouses can you build?

ANSWER $3 \cdot 6 = 18$, so you can build 18 different birdhouses.

2. You have 5 designs of birdhouses and 4 colors from which to choose. How many different birdhouses can you build?

12.5–12.6 Can you find permutations and combinations?

Review

EXAMPLE At a swim meet, 10 swimmers are in an event. In how many ways can first, second, third, and fourth place medals be awarded?

The order is important, so find the number of permutations.

$$_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

ANSWER There are 5040 ways to award the medals.

3. You are making a braided rope out of three different colors of yarn. You have seven colors of yarn. In how many ways can you choose the colors so the rope has three different colors?

12.7 Can you find odds?

Review

EXAMPLE You have a bag of 6 red, 5 blue, and 3 white marbles. What are the odds in favor of randomly drawing a red marble from the bag?

ANSWER Odds $= \dfrac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}} = \dfrac{6}{8} = \dfrac{3}{4}$

☑ **4.** What are the odds in favor of randomly drawing a white marble ?

12.8 Can you calculate probabilities?

Review

EXAMPLE Tiles with each of the 11 letters in PROBABILITY are in a bag. You randomly draw a tile, replace it, and then randomly draw a second tile. What is the probability that both tiles are I's?

ANSWER $P(\text{I and I}) = P(\text{I}) \cdot P(\text{I}) = \dfrac{2}{11} \cdot \dfrac{2}{11} = \dfrac{4}{121}$

☑ **5.** What is the probability that the first tile is B and the second is L?

Stop and Think about Lessons 12.4–12.8

6. Writing Give a real-world example of a situation where you use combinations to count possibilities.

Review Quiz 2

1. **Shoes** You can buy sandals or sneakers in black, brown, tan, or white. Make a tree diagram to show the possible choices for shoes.
2. **Camp** You are scheduling swimming, crafts, canoeing, and softball. How many different schedules of four different activities are possible?
3. **Hockey** Find the number of ways two players can be chosen from 20 team members.
4. **Rain** If the probability that it will rain today is 0.4, what are the odds in favor of rain? What is the probability that it will *not* rain?

Find the probability.

5. You roll a 5 on a 6-sided number cube. Then you roll another 5.
6. A bag has 3 red and 5 blue tiles. You randomly draw a red tile, keep it, and then randomly draw a blue tile.

CHAPTER 12

Chapter Review

Vocabulary

stem-and-leaf plot, p. 597
box-and-whisker plot, p. 601
lower quartile, p. 601
upper quartile, p. 601
lower extreme, p. 601
upper extreme, p. 601
circle graph, p. 605
line graph, p. 606
tree diagram, p. 618
permutation, p. 623
factorial, p. 623
combination, p. 627
complementary events, p. 632
unfavorable outcome, p. 633
odds, p. 633
independent events, p. 639
dependent events, p. 639

Vocabulary Review

Matching **In Exercises 1–6, match the description with the correct word(s).**

1. An arrangement in which order is important

2. An arrangement in which order is not important

3. A graph to display data that fall into distinct categories

4. A graph used to display changes in a quantity over time

5. A plot used to order a data set

6. A plot used to summarize a data set

A. bar graph

B. line graph

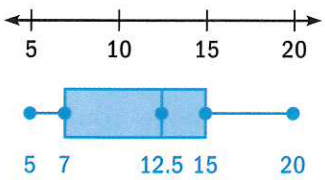
C. box-and-whisker plot

D. stem-and-leaf plot

E. combination

F. permutation

7. Copy the box-and-whisker plot at the right. Label the median, upper and lower quartiles, and upper and lower extremes.



Review Questions

Make an ordered stem-and-leaf plot to organize the data. Identify the interval that includes the most data values. *(Lesson 12.1)*

8. 20, 25, 36, 16, 29, 32, 27, 42

9. 11.2, 7.5, 15.1, 15.7, 15.0, 6.7, 11.3

10. **Chess** The prices of several chess sets are \$15, \$20, \$38, \$95, \$60, \$45, \$40, \$35, and \$50. Make a box-and-whisker plot of the data. What conclusions can you make? *(Lesson 12.2)*

Review Questions

11. Summer Treats The table shows the favorite summer treats of students surveyed. Represent the data in a circle graph. *(Lesson 12.3)*

Ice Cream	50%
Frozen fruit	25%
Ices	10%
Other	15%

12. Election Your class is having an election for president, vice president, and secretary. For president there are 4 candidates, for vice president there are 5 candidates, and for secretary there are 3 candidates. No one is running for more than one office. How many groups of winners are possible? *(Lesson 12.4)*

Evaluate the expression. *(Lessons 12.5, 12.6)*

13. ${}_8P_4$ **14.** ${}_{10}P_3$ **15.** ${}_9C_2$ **16.** ${}_6C_1$

17. Photograph You and six friends are posing for a photograph. In how many ways can you line up for the photograph if you line up in one row? *(Lesson 12.5)*

In Exercises 18 and 19, tell whether the situation describes a *combination* or a *permutation*. Then find the answer. *(Lessons 12.5, 12.6)*

18. Pizza In how many ways can you select 4 different pizza toppings from 12 toppings?

19. Bobblehead Dolls You have six different bobblehead dolls, and you want to choose three to give as gifts to Ali, Lin, and Rhea. How many different ways can you do this?

20. Contest The probability that you will win a contest is 76%. What is the probability that you will lose the contest? What are the odds that you will lose the contest? *(Lesson 12.7)*

21. Softball A softball player has a batting average of 0.350, which means she gets a hit 35% of her times at bat. What are the odds that she will get a hit in her next at bat? *(Lesson 12.7)*

22. Weather The weather forecaster says there is a 60% chance that it will snow on Wednesday and a 25% chance that it will snow on Thursday. Find the probability that it will snow on both Wednesday and Thursday. *(Lesson 12.8)*

23. Cards Two cards are dealt randomly, one after another, from a deck of cards numbered from 1 through 20. The first card is not returned to the deck before the second is dealt. Find the probability that the first card is a 7 and the second card is a 4. *(Lesson 12.8)*

CHAPTER 12

Chapter Test

In Exercises 1 and 2, make a stem-and-leaf plot. Tell which interval includes the most values. Then make a box-and-whisker plot.

1. 46 kg, 70 kg, 21 kg, 136 kg, 55 kg, 60 kg, 72 kg, 104 kg, 52 kg

2. 12.1 in., 13.5 in., 12.8 in., 10 in., 7 in., 11.2 in., 12.9 in., 11.1 in., 12 in., 13.7 in.

3. **Academy Awards** The lengths, in minutes, of the Best Picture Academy Award winning movies for the years 1990–1999 are 99, 118, 122, 131, 142, 160, 177, 183, 194, and 197. Make a box-and-whisker plot of the data.

4. **Temperature** The record low temperatures in Miami, Florida, are given in the table. Display the data in an appropriate graph.

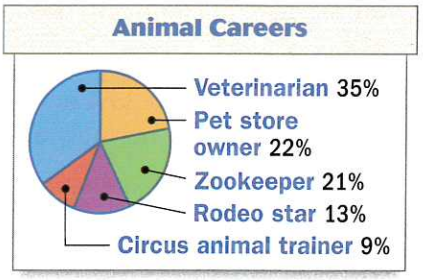
Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Temp (°F)	30	32	32	46	53	60	69	68	68	51	39	30

5. **Golf** There are five members on a golf team. Make a tree diagram to count the number of ways you can select a captain and an assistant.

6. **Movies** You and four friends are going to a movie. In how many different orders can you pick your friends up? List all the possible orders.

7. **School Dance** You are making a banner for a school dance and have a choice of 8 colors. You want to use 4 different colors. How many different combinations are possible?

In Exercises 8 and 9, use the circle graph. It shows student replies to *Which animal career would you enjoy?*



8. What is the probability that a randomly chosen student replied *veterinarian*?

9. What are the odds in favor of a randomly chosen student replying *zookeeper*?

Tell whether the events are *independent* or *dependent*. Then find the probability.

10. You roll a 6 on a number cube. Then you roll again and roll a 2.

11. You have 8 blue marbles and 12 red marbles in a bag. You randomly pick a blue marble on the first draw. Then you randomly pick another blue marble without replacing the first one.

Chapter Standardized Test

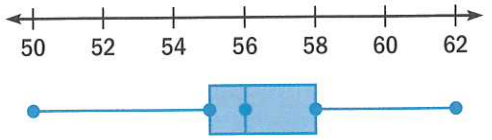
Test-Taking Strategy **Learn as much as you can about a test ahead of time, such as the types of questions and the topics the test will cover.**

Multiple Choice

1. When the number 29 is plotted on a stem-and-leaf plot, the 9 is which of the following?

A. stem **B.** leaf **C.** key **D.** median

2. What does the number 56 represent on the box-and-whisker plot?



F. upper quartile **G.** median

H. lower quartile **I.** lower extreme

3. Which is an appropriate display for the data in the table below?

Favorite sport	Percent of students
Football	17%
Baseball	28%
Basketball	39%
Soccer	8%

A. bar graph **B.** line graph

C. circle graph **D.** histogram

4. You are asked to enter a 4-character password for a video game. The password must begin with a letter and end with 3 digits. How many different passwords are possible if you can repeat digits?

F. 56 **G.** 26,000

H. 175,760 **I.** 456,976

5. An ice cream parlor has 8 different flavors of ice cream. You would like a dish with 3 scoops of different flavors. How many different dishes can you pick?

A. 36 **B.** 56

C. 336 **D.** 40,320

6. There are 200 raffle tickets and 5 are winning tickets. What are the odds in favor of winning with one ticket?

F. 1 to 40 **G.** 1 to 39

H. 1 to 199 **I.** 1 to 200

7. You pick randomly from a jar of 12 green, 18 yellow, and 20 red mints. You pick a mint, eat it, and pick another mint. What is the probability that you pick a green mint and then a yellow mint?

A. $\frac{99}{1225}$ **B.** $\frac{54}{625}$ **C.** $\frac{108}{1225}$ **D.** $\frac{27}{152}$

Short Response

8. There are 30 students auditioning for new openings in a chorus. How many ways can you choose 4 students to be in the chorus?

Extended Response

9. You asked 100 students whether they had shirts of the following colors: blue, yellow, orange. Your results were blue: 94%, yellow: 68%, orange: 43%. Use an appropriate form to display these data. Explain your choice. Then identify another type of display that would not be a good choice for these data. Explain why it would not be a good choice.

INVESTIGATING Robins

Geometric Probability

When a robin hunts for worms, does it search randomly, or can it sense a worm's location? Scientists used geometric probability to investigate this. Geometric probability is based on area. For events dependent on area, you can find the *geometric probability* using the following formula:

$$P(\text{event}) = \frac{\text{Area representing favorable outcomes}}{\text{Area representing possible outcomes}}$$

There are 4 treasure chests buried in a 5 yard by 10 yard field. Each chest has an area of 1 square yard. Use geometric probability to describe the expected results of searching a randomly chosen spot in the field.

(1) Find the area of the treasure chests.

$$4 \text{ chests} \cdot \frac{1 \text{ square yard}}{\text{chest}} = 4 \text{ square yards}$$

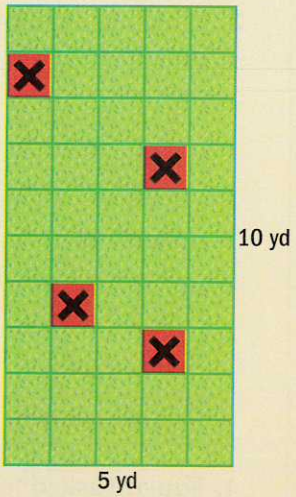
(2) Find the area of the field.

$$5 \text{ yards} \cdot 10 \text{ yards} = 50 \text{ square yards}$$

(3) Find the geometric probability of finding a treasure chest by random search.

$$P(\text{finding a chest}) = \frac{\text{Area of treasure chests}}{\text{Area of field}}$$

$$= \frac{4}{50} = 0.08$$



So, the geometric probability of finding a chest by random search is 8%.

1. Suppose there are 6 treasure chests in the field. What is the geometric probability of finding a chest by random search?
2. Suppose there are 9 treasure chests in the field, and each has an area of 2 square feet. Find the geometric probability of finding a chest by random search. Remember to measure the field in the same units as the chests.

How Robins Find Worms

To investigate how robins find worms, scientists did a series of experiments. They buried four worms in pans of soil that were marked into a 10 by 10 grid and calculated the geometric probability of a robin finding a worm at random.

Then they let robins search the pan and recorded the percent of the time a worm was found. A robin was counted as finding a worm if the square that it searched contained a worm. The experiment was repeated under different conditions.

Condition	Correct Finds
Robins could use all senses.	90%
Robins prevented from using visual clues.	50%
White noise decreased robins' ability to hear worms.	59%

3. Find the geometric probability of finding a worm by searching randomly. Write the probability as a percent.
4. The table above shows the percent of attempts in which the robins found worms under each condition. For each condition, compare the percent of correct finds to the geometric probability of randomly finding a worm.
5. **Critical Thinking** What can you conclude from the results of the experiment? How do the results suggest that robins don't search randomly for worms but instead use their senses?

Project IDEAS

- **Experiment** Design and perform an experiment or a search involving geometric probability. Compare the theoretical and experimental geometric probabilities. Explain any discrepancies. Present your findings to the class.
- **Investigate** Learn more about how different birds find food. Present your findings.
- **Research** Look up information about search and rescue techniques. What techniques do people use to increase the probability of finding something? Present your findings.
- **Career** Learn about people who study animal behavior. What sorts of careers do they have? Present your findings.

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CHAPTER 13

Polynomials and Functions

BEFORE

In previous chapters you've...

- Simplified expressions by combining like terms
- Graphed linear functions

Now

In Chapter 13 you'll study...

- Simplifying polynomials
- Adding and subtracting polynomials
- Multiplying binomials
- Graphing non-linear functions

WHY?

So you can solve real-world problems about...

- treehouses, p. 663
- baseball, p. 676
- stage design, p. 677
- juggling, p. 680

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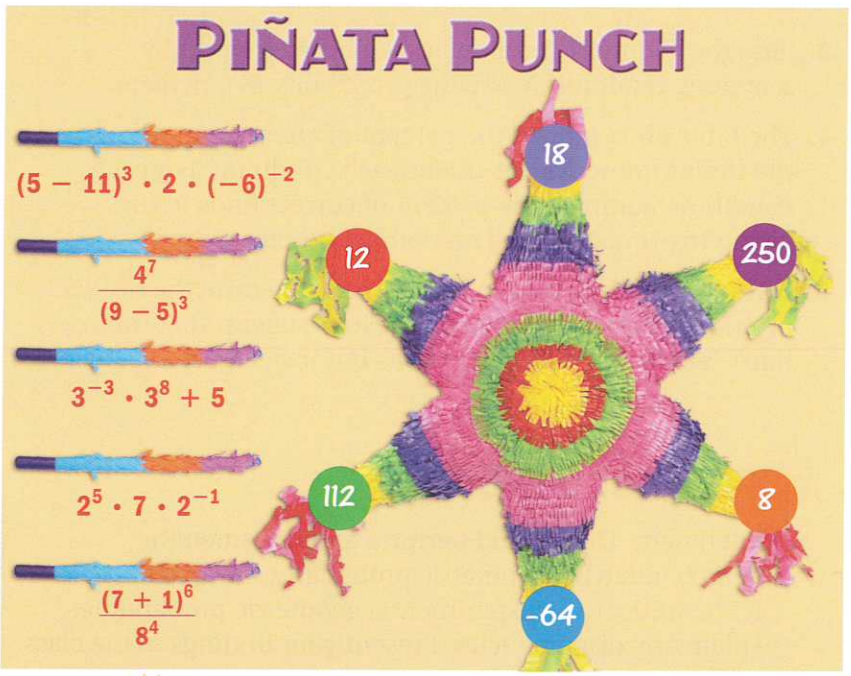
Chapter Warm-Up Games

Review skills you need for this chapter in these quick games.

PIÑATA PUNCH



BRAIN GAME

Key Skill:
Using properties of exponents

To break open the piñata on your turn, you need to pick the right stick.

- Evaluate the expression under each stick.
- A stick breaks the piñata if it has the same value as one of the spots.
- Which stick breaks the piñata? Which spot should you hit?