CHAPTER 13

Polynomials and Functions

BEFORE

In previous chapters you've...

- Simplified expressions by combining like terms
- Graphed linear functions

NOW

In Chapter 13 you'll study...

- Simplifying polynomials
- Adding and subtracting polynomials
- Multiplying binomials
- Graphing non-linear functions

WHY?

So you can solve real-world problems about...

- treehouses, p. 663
- baseball, p. 676
- stage design, p. 677
- juggling, p. 680

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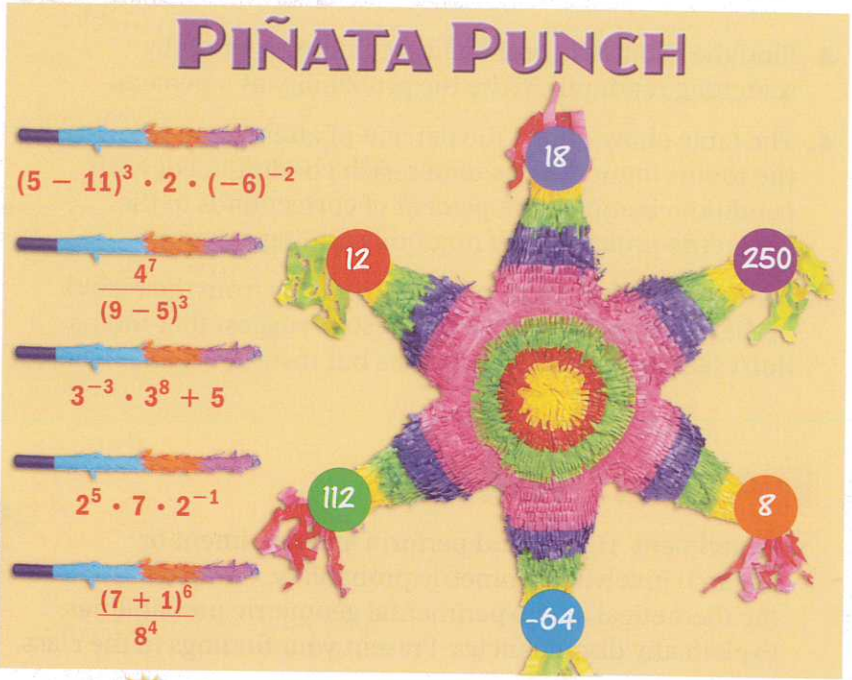
- More Examples

Chapter Warm-Up Games

Review skills you need for this chapter in these quick games.

PIÑATA PUNCH



BRAIN GAME

Key Skill:
Using properties of exponents

To break open the piñata on your turn, you need to pick the right stick.

- Evaluate the expression under each stick.
- A stick breaks the piñata if it has the same value as one of the spots.
- Which stick breaks the piñata? Which spot should you hit?

How Robins Find Worms

To investigate how robins find worms, scientists did a series of experiments. They buried four worms in pans of soil that were marked into a 10 by 10 grid and calculated the geometric probability of a robin finding a worm at random.

Then they let robins search the pan and recorded the percent of the time a worm was found. A robin was counted as finding a worm if the square that it searched contained a worm. The experiment was repeated under different conditions.

Condition	Correct Finds
Robins could use all senses.	90%
Robins prevented from using visual clues.	50%
White noise decreased robins' ability to hear worms.	59%

3. Find the geometric probability of finding a worm by searching randomly. Write the probability as a percent.
4. The table above shows the percent of attempts in which the robins found worms under each condition. For each condition, compare the percent of correct finds to the geometric probability of randomly finding a worm.
5. **Critical Thinking** What can you conclude from the results of the experiment? How do the results suggest that robins don't search randomly for worms but instead use their senses?

Project IDEAS

- **Experiment** Design and perform an experiment or a search involving geometric probability. Compare the theoretical and experimental geometric probabilities. Explain any discrepancies. Present your findings to the class.
- **Investigate** Learn more about how different birds find food. Present your findings.
- **Research** Look up information about search and rescue techniques. What techniques do people use to increase the probability of finding something? Present your findings.
- **Career** Learn about people who study animal behavior. What sorts of careers do they have? Present your findings.

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Unmasking Expressions

$x - 4x + 3 + 9x$

$-7x + 8 - 2(3x + 4)$

$5(x - 6) + 10x - 3$

$8x - 3 - 4(2x + 3)$

$4 - (3x - 1) + x$

$17 - 4x - 13 + 2x$

John	Maria	Sam	Carol	Vincent	Sophie
$6x + 3$	$-2x + 4$	$-13x$	-15	$-2x + 5$	$15x - 33$

BRAIN GAME

Key Skill:
Combining like terms

Find who is behind each of the masks.

- Match the expression under the mask with the correct simplified expression below a name.

Stop *and* Think

1. **Writing** Explain the steps for simplifying the expression $-7x + 8 - 2(3x + 4)$.
2. **Critical Thinking** Write $2^5 \cdot 7 \cdot 2^{-1}$ as an expression with only positive exponents.

CHAPTER 13

Getting Ready to Learn

Review What You Need to Know

Using Vocabulary **Copy and complete with a review word.**

1. The ? of $2x^3$ is 2.
2. A(n) ? is a relation that assigns exactly one output value to each input value.
3. A number, a variable, or a product of a number and one or more variables is a(n) ?.

Word Watch

Review Words

power, p. 20
exponent, p. 20
like terms, p. 86
coefficient, p. 86
monomial, p. 170
function, p. 541

Simplify the expression. *(p. 85)*

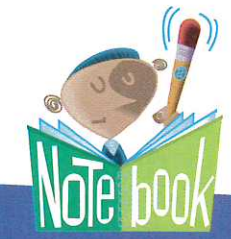
4. $6x - 4 + 4x - 3$
5. $-5(2x + 3) - 4x$
6. $7(3x - 5) - (-x)$
7. $-2x - (-5x)$
8. $-2(-4x - 8)$
9. $-3(3x) + 18x$

Simplify the expression. *(p. 196)*

10. $\frac{y^4}{y^6}$
11. $\frac{5^{37}}{5^{35}}$
12. $x^4 \cdot x^5$

List four solutions of the equation. *(p. 550)*

13. $y = 3x - 5$
14. $y = -2x + 1$
15. $y = \frac{1}{2}x$



You should include material that appears on a notebook like this in your own notes.

Know How to Take Notes

Summarizing Material Summarize the main ideas from different lessons in your notebook. This will help you to see how key ideas are related.

Exponent Rules

Product of Powers

$x^2 \cdot x^3 = x^{2+3}$

$= x^5$

Quotient of Powers

$\frac{x^5}{x^2} = x^{5-2}$

$= x^3$

Zero Exponent

for $x \neq 0$,

$x^0 = 1$

Negative Exponents

for $x \neq 0$,

$x^{-4} = \frac{1}{x^4}$

In Lesson 13.4, you can summarize key ideas about algebra in your notebook.

LESSON 13.1

Polynomials

BEFORE You simplified expressions by combining like terms.

NOW You will simplify polynomials by combining like terms.

WHY? So you can find the height of a falling pinecone, as in Example 3.

Word Watch

polynomial, p. 657
binomial, p. 657
trinomial, p. 657
standard form, p. 657

A **polynomial** is a monomial or a sum of monomials. Each monomial in a polynomial is called a *term*. Polynomials are classified by the number of their terms. If a polynomial has more than three terms, it is simply called a polynomial.

Monomial (1 term)	Binomial (2 terms)	Trinomial (3 terms)
$-2x$	$3x - 2$	$-2a^2 + 3a + 1$
4	$-s^4 + 6s^3$	$3 + 5r - 7r^2$

A polynomial is written in **standard form** if the exponents of the variable decrease from left to right.

Standard Form	Not Standard Form
$3x^3 - 2x^2 + 4$	$3 + 5y$
$-2m^6 + 5m^3 - m$	$7t^4 - t^7 - 2t^2 + 3t$

EXAMPLE 1 Writing Polynomials in Standard Form

Write the polynomial in standard form. Classify the polynomial.

a. $x - 9 + 5x^2$

$= x + (-9) + 5x^2$ Write subtraction as addition.

$= 5x^2 + x + (-9)$ Order terms with decreasing exponents.

ANSWER The polynomial $5x^2 + x - 9$ has 3 terms, so it is a trinomial.

b. $2x - 3x^3$

$= 2x + (-3x^3)$ Write subtraction as addition.

$= -3x^3 + 2x$ Order terms with decreasing exponents.

ANSWER The polynomial $-3x^3 + 2x$ has 2 terms, so it is a binomial.

Watch Out!

If you do not see an exponent with a variable, then its exponent is 1.

$2x = 2x^1$

Your turn now **Write the polynomial in standard form and classify it.**

1. $4 + b^2 - 8b$ **2.** $-5 + 3x^2$ **3.** $11 + 2n^4 - 7n + 5n^2$

Simplifying Polynomials Remember that *like terms* have the same variables raised to the same powers. To simplify a polynomial, combine like terms by adding their coefficients.

EXAMPLE 2 Simplifying Polynomials

Simplify the polynomial and write it in standard form.

a. $3x^2 + 4x^2 - 2x - 3$

$= (3x^2 + 4x^2) - 2x - 3$ Group like terms.

$= 7x^2 - 2x - 3$ Simplify.

b. $x^2 + 2 + 4(x^2 - 2x)$

$= x^2 + 2 + 4x^2 - 8x$ Use the distributive property.

$= (x^2 + 4x^2) + 2 - 8x$ Group like terms.

$= 5x^2 + 2 - 8x$ Simplify.

$= 5x^2 - 8x + 2$ Write in standard form.

HELP with Review

Remember that $x^2 = 1x^2$. For help with like terms, see p. 85.

Your turn now **Simplify the polynomial and write it in standard form.**

4. $7p + 5p^2 - 2 - 3p^2$

5. $10s^4 - 3s + s^4 - 1$

6. $2(a^2 + 3a - 1) + 2a^2$

7. $8x + 3(2x^2 - x + 1)$

EXAMPLE 3 Evaluating a Polynomial Expression

Pinecone You drop a pinecone from a 150 foot bridge. The height of the pinecone, in feet, after t seconds of falling, can be found using the polynomial $-16t^2 + 150$. Find the pinecone's height after 2 seconds.

Solution

$-16t^2 + 150 = -16(2)^2 + 150$ Substitute 2 for t.

$= -16(4) + 150$ Evaluate the power.

$= -64 + 150$ Multiply.

$= 86$ Add.

ANSWER The pinecone's height after 2 seconds is 86 feet.

Your turn now **Find the height of the pinecone in Example 3 after it falls for the given number of seconds.**

8. 0.5 sec **9.** 1 sec **10.** 1.5 sec **11.** 3 sec

13.1 Exercises

More Practice, p. 739

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Getting Ready to Practice

Vocabulary **Classify the polynomial as a *monomial*, a *binomial*, or a *trinomial*.**

1. $x^2 + 3x - 7$ **2.** $y - 5$ **3.** $8s^2t$ **4.** $2a^2 + 9a^3 + a$

Write the polynomial in standard form.

5. $7 + 3m$ **6.** $5n - 1 - n^2$ **7.** $4b - 4 + 6b^3$

Simplify the polynomial and write it in standard form.

8. $3x + x^2 - 2x$ **9.** $4 + 5y - 5$ **10.** $-9 + 7m^3 - 2m^3$

11. Find the Error Describe and correct the error in simplifying the polynomial.

$$-3x^2 - 4(5x + 1)$$
$$= -3x^2 - 20x - 4$$
$$= -23x - 4$$

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	12–14
2	15–20, 23–26
3	27–30

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Write the polynomial in standard form. Classify the polynomial.

12. $2 - 5y + y^2$ **13.** $-13x^3 + 4x^{10}$ **14.** $3 - r^4 + r + 2r^3$

Simplify the polynomial and write it in standard form.

15. $3x - 4 + x$

16. $2c^2 - c^2 + 5c$

17. $4q^3 - 7q^5 + 3q - q^3$

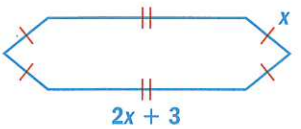
18. $7 - 4d^2 - 3d^2 + d$

19. $12 + 3b - 6b + 5$

20. $g^3 - 10 + 2g^2 - 5g^3$

Measurement **Write a polynomial expression for the perimeter. Simplify the polynomial and write it in standard form.**

21.



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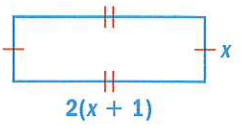


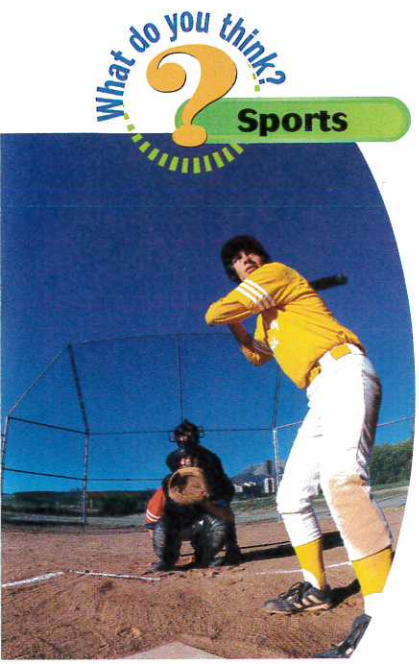
Simplify the polynomial and write it in standard form.

23. $1 + 12m^2 - 5m + 6m - 7$

24. $7 - p^3 - 5p^3 + 3p + p - p^3$

25. $3x^2 + 5(x^2 - 3x + 6)$

26. $-6(2y^3 - 4y^2 + 1) + 10y^2$



Baseball

A player hits a ball 60 mi/h into right field. The ball has the same height after 2 seconds as it has after 3.5 seconds. How is this possible?

Baseball **A player hits a ball 60 mi/h, or 88 ft/sec, toward right field. Evaluate the polynomial $-16t^2 + 88t + 2$ to find the ball's height, in feet, after t seconds.**

27. $t = 1.5$ **28.** $t = 2$ **29.** $t = 2.5$ **30.** $t = 3$

Simplify the polynomial and write it in standard form.

31. $-4(t - 3t^2 + 8 - 4t) + 6t^2 - 5$ **32.** $-3(-s^4 + 2s - 6 - s) - 8s + s^4$

Critical Thinking **Tell whether the statement is *always*, *sometimes*, or *never* true.**

33. The terms of a trinomial are monomials.

34. A monomial has one factor.

35. A binomial has more than two terms.

Challenge **Simplify the polynomial.**

36. $3x^2 - 2y + 5x^2 - 4$ **37.** $-16t - 7h + 3t^2 + 4h$

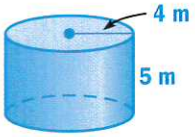
38. $5a - 4(3b + 6) + 4b$ **39.** $-z^2 + 3z - 2(4y - 5z)$

40. Make a Connection Find the meaning of the prefix *poly*. Explain what this tells you about the words *polygon* and *polynomial*.

Mixed Review

41. The letters in the word FUNCTION are put in a bag. Find the probability of drawing the letter N at random. *(Lesson 7.8)*

42. Find the volume of the cylinder. Round to the nearest hundredth of a cubic meter. *(Lesson 10.6)*



43. Make a stem-and-leaf plot of the data. *(Lesson 12.1)*
38.6, 35.8, 36.3, 34.2, 37.6, 37.5, 36.4, 36.2, 38.6, 36.6

Basic Skills **Evaluate the expression.**

44. $8(3 + 9)$ **45.** $-3(11 - 4)$ **46.** $-5(7 + 2)$ **47.** $12(6 - 1)$

Test-Taking Practice

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48. Multiple Choice Simplify the polynomial $5(x^2 - 2x - 3) - 9x^2$.

A. $4x^2 + 10x - 15$ **B.** $-4x^2 + 10x - 15$

C. $-4x^2 - 15x - 10x$ **D.** $-4x^2 - 10x - 15$

49. Multiple Choice Find the value of $-2x^2 - 4x + 7$ when $x = -2$.

F. -9 **G.** 7 **H.** 15 **I.** 23

LESSON 13.2

Adding and Subtracting Polynomials

BEFORE You simplified polynomials.

NOW You'll add and subtract polynomials.

WHY? So you can find the area of clay coasters, as in Ex. 29.

Word Watch

Review Words

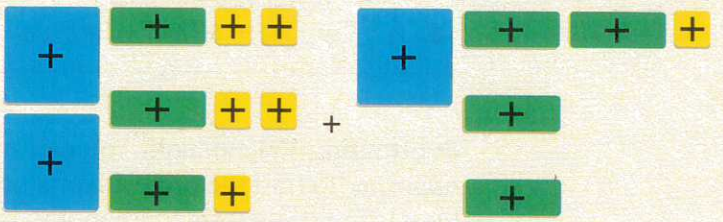
opposite, p. 54
like terms, p. 86

Activity You can model polynomial addition with algebra tiles.

You can model polynomials with algebra tiles.

x^2-tile x-tile 1-tile

(1) Write the two polynomials represented by the algebra tiles.



(2) Group the algebra tiles to model the sum of the polynomials. Draw your model. Write the polynomial that your drawing represents.

(3) Use algebra tiles to model the sum of the polynomials below. Write the polynomial that your model represents.

a. $(3x^2 + 6x + 1) + (x^2 + x)$ **b.** $(2x^2 + 3x + 1) + (4x^2 + x)$

In the activity, you used algebra tiles to add two polynomials. You add polynomials by combining like terms.

EXAMPLE 1 Adding Polynomials Vertically

Find the sum $(-4x^3 + x^2 - 3x - 1) + (4x^2 - 7x + 5)$.

Solution

$$\begin{array}{rl} & -4x^3 + x^2 - 3x - 1 \\ + & \quad 4x^2 - 7x + 5 \\ \hline & -4x^3 + 5x^2 - 10x + 4 \end{array}$$

Write the second polynomial under the first.

Arrange like terms in columns.

Add like terms.

In Example 1, you combined like terms vertically. You can also add polynomials by combining like terms horizontally.

HELP with Solving

When you regroup terms, you must move a subtraction or addition sign with the term that follows it.

EXAMPLE 2 Adding Polynomials Horizontally

Find the sum $(2y^2 - 4y + 6) + (y^2 + 3y - 2)$.

Solution

$$(2y^2 - 4y + 6) + (y^2 + 3y - 2)$$

$= 2y^2 + y^2 - 4y + 3y + 6 - 2$ Group like terms.

$= 3y^2 - y + 4$ Combine like terms.

Your turn now **Find the sum.**

1. $(6x^2 - 3x + 1) + (3x^2 + 4x - 5)$
2. $(5n^2 + 2n - 9) + (3n^2 - n + 4)$
3. $(y^2 - y + 1) + (-2y^2 + 2y - 1)$
4. $(3p^2 - p - 1) + (p^2 + p - 4)$

Subtracting Polynomials You can subtract a polynomial by adding its *opposite*. To find the opposite of a polynomial, multiply each of its terms by -1. You can subtract polynomials vertically or horizontally.

EXAMPLE 3 Subtracting Polynomials Vertically

Find the difference $(4x^3 + 5x^2 - 2x - 5) - (3x^3 - 4x + 2)$.

Solution

(1) Find the opposite of the second polynomial.

$$-(3x^3 - 4x + 2) = -3x^3 + 4x - 2$$

(2) Find the sum $(4x^3 + 5x^2 - 2x - 5) + (-3x^3 + 4x - 2)$.

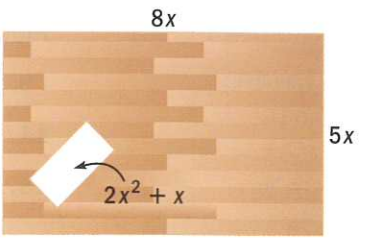
$$\begin{array}{rr} 4x^3 + 5x^2 - 2x - 5 & \text{Write the second polynomial under the first.} \\ \underline{+\ -3x^3 \qquad + 4x - 2} & \text{Arrange like terms in columns.} \\ x^3 + 5x^2 + 2x - 7 & \text{Add like terms.} \end{array}$$

Your turn now **Find the difference.**

5. $(4r^2 - r + 8) - (r^2 + 6r - 1)$
6. $(6m^2 + 2m - 3) - (7m^2 + 4)$
7. $(5t^2 + 4t + 1) - (2t^2 + 8t + 11)$
8. $(x^2 + 5x + 7) - (3x^2 - 4x - 2)$

EXAMPLE 4 Finding the Area of a Tree House

Tree House The design for a tree house calls for a rectangular hole in the floor. Write a polynomial expression for the area of the tree house floor.



Solution

To find the area of the floor, use the area of the two rectangles.

Area of Large Rectangle $8x \cdot 5x = 40x^2$

Area of Small Rectangle $2x^2 + x$

Area of floor	=	Area of large rectangle	−	Area of small rectangle

$= 40x^2 - (2x^2 + x)$

$= 40x^2 - 2x^2 - x$ **Distributive property**

$= 38x^2 - x$ **Combine like terms.**

ANSWER A polynomial expression for the area of the floor is $38x^2 - x$.

13.2 Exercises

More Practice, p. 739

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Getting Ready to Practice

1. Vocabulary Copy and complete: To add polynomials, you should combine ?.

Find the sum or difference.

2. $(8y + 5) + (4y - 3)$

3. $(7x + 10) - (x - 2)$

4. $(x - 6) + (2x + 9)$

5. $(4p + 1) - (p - 7)$

6. $(5n^2 + 2n + 1) - (4n^2 - 1)$

7. $(-3a + 10) + (2a - 4)$

8. Find the Error Describe and correct the error(s) in the solution.

$$\begin{array}{r} -4x^3 + 5x^2 - 7x + 2 \\ +\ 2x^3 - 6x + 10 \\ \hline -2x^3 - x^2 + 3x + 2 \end{array}$$

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	9–14
2	9–14
3	15–20
4	29

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Find the sum.

9. $(4x + 7) + (x - 3)$

10. $(-2a - 9) + (a + 4)$

11. $(3n - 7) + (4n + 5)$

12. $(t^2 + 3t) + (3t^2 + 8t)$

13. $(-g^2 + g + 9) + (7g^2 - 6)$

14. $(3z^2 - 2z + 1) + (4z^3 + 3z)$

Find the difference.

15. $(-5d - 1) - (5d + 6)$

16. $(7y + 1) - (3y - 2)$

17. $(2h^2 + 9h) - (13h^2 - h)$

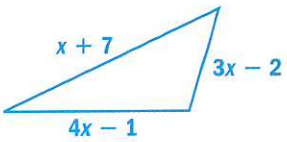
18. $(4x^2 + 9x) - (x^2 + 7x - 1)$

19. $(6r^2 + 2r - 5) - (3r^2 - 9)$

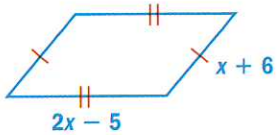
20. $(-4b^3 - 9b + 2) - (b^3 - b + 3)$

Geometry **Write a polynomial expression for the perimeter of the figure. Simplify the polynomial.**

21.



22.



Find the sum or difference.

23. $(2k^2 + 5k) + (4k^2 - 5k)$

24. $(5a^2 + 3a + 8) - (2a^2 - 2a - 9)$

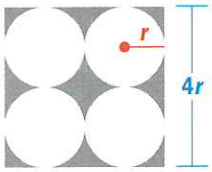
25. $(6x^3 - 12x + 1) + (8x^2 - 4)$

26. $(4p^3 + p^2 - 8) - (7p^3 + 2p + 5)$

27. $(4n - 3) + (9n + 5) - (n - 1)$

28. $(-8m + 1) - (2m - 6) + 5m$

29. Coasters To make a set of coasters, you cut identical circles out of a square piece of clay. Write a polynomial expression for the area of clay that remains after you remove the circles. Simplify the polynomial. Is there enough clay left over to make another coaster of the same radius and thickness? Explain your answer.



Perform the indicated operations.

30. $-2(5y + 3) - 9(y + 1)$

31. $4(-3s^2 + s - 4) + (5s^2 + s + 7)$

32. $3(q^2 - q) + 2(7q^2 - 2q)$

33. $6(t^3 - t^2 + 3t) - 4(5t^3 + t^2 - t)$

34. $5(4x^3 - 2x^2 + 1) + 3(7x^2 - 5x)$

35. $-7(2v^4 + 3v^2 - 1) - 5(-3v^3 - 6)$

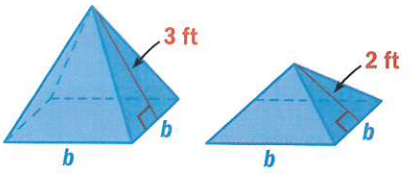
36. Critical Thinking Can the sum of two trinomials be a binomial? Give an example to justify your answer.

37. Challenge Solve the equation $(2x^2 - 3x + 4) - (2x^2 + x - 8) = 0$.

HELP with Review

For help with surface area of a pyramid, see p. 507.

38. Science Fair You are constructing two wooden pyramids using the designs shown. Write a simplified polynomial expression for the total surface area of the two pyramids.



Mixed Review

Simplify the expression. *(Lesson 4.6)*

39. $b^3 \cdot b^7$ **40.** $\frac{x^{12}}{x^5}$ **41.** $\frac{m^4n^5}{n^2}$ **42.** $\frac{a^2 \cdot a^6}{a^3}$

43. How many different passwords can be made using 4 digits from 0 to 9? *(Lesson 12.4)*

Test-Taking Practice

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44. Multiple Choice Find the sum $(-7x^3 + 4x^2 - 1) + (x^3 - 9x + 3)$.

A. $-6x^3 + 4x^2 - 9x + 2$ **B.** $-8x^3 + 4x^2 - 9x - 4$

C. $-6x^3 + 5x^2 + 2$ **D.** $-8x^3 - 5x^2 + 4$

45. Short Response Explain how to find the difference.

$(x^4 - 3x^3 + 5x + 3) - (x^4 + 2x^3 - 9x^2)$

BRAIN GAME

Polynomial Potions

You need to make six potions using the six ingredients in the laboratory. Each potion is made by adding two ingredients together. Use the list of ingredients and the potion labels to find the secret formulas.

Ingredients

Dog Biscuits
$x^2 - 2x + 2$

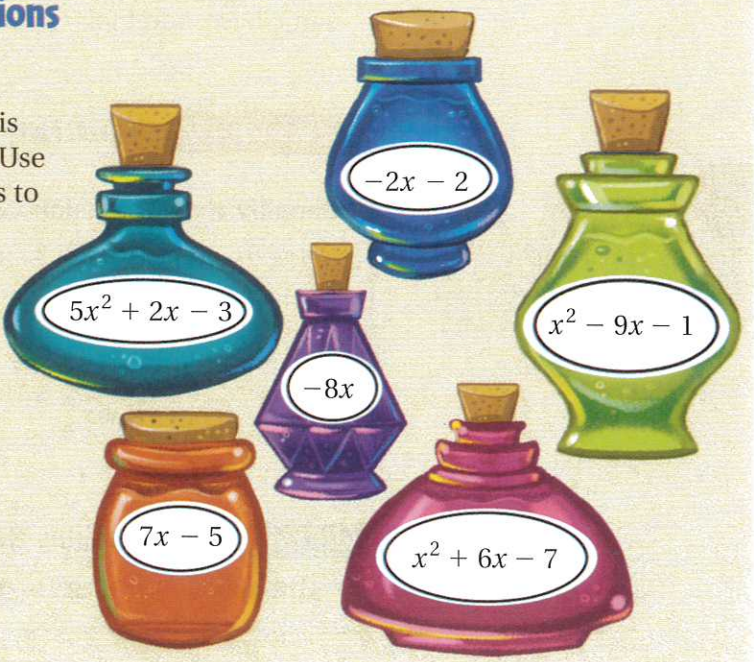
Cotton Balls
$-x^2 - 6x - 2$

Green Slime
$2x^2 - 3x + 1$

Muck
$-3x^2 + 2x - 1$

Fish Oil
$3x^2 + 5x - 4$

Nail Polish
$-2x^2 + x - 3$



LESSON 13.3

Monomials and Powers

BEFORE You added and subtracted polynomials.

NOW You will apply properties of exponents to monomials.

WHY? So you can find the volume of Saturn, as in Ex. 48.

Word Watch

Review Words

power, p. 20
exponent, p. 20
coefficient, p. 86
monomial, p. 170

Activity **You can use the properties of exponents to simplify monomials.**

(1) Copy and complete the table by expanding each expression, regrouping factors, and simplifying.

Expression	Expand	Regroup	Simplify
$(3x)(4x^2)$	$3 \cdot x \cdot 4 \cdot x \cdot x$	$3 \cdot 4 \cdot x \cdot x \cdot x$	$12x^3$
$(-2x)(5x^4)$	?	?	?
$(xy)^3$	$xy \cdot xy \cdot xy$	$x \cdot x \cdot x \cdot y \cdot y \cdot y$	x^3y^3
$(4x)^2$	?	?	?
$(-3x)^3$	?	?	?

(2) What patterns do you notice in the table?

(3) Use your results to simplify the expressions $(5x)(2x^3)$ and $(3pq)^2$.

In the activity, you used properties of exponents that you learned in Lesson 4.6 to multiply monomials. To multiply factors that have the same base, add their exponents. Multiply their coefficients.

HELP with Review

For help with rules of exponents, see p. 196.

EXAMPLE 1 Multiplying Monomials

Simplify the expression $(2x^3)(-3x)$.

$$(2x^3)(-3x) = 2 \cdot x^3 \cdot (-3) \cdot x \qquad \text{Expand the expression.}$$
$$= 2 \cdot (-3) \cdot x^3 \cdot x \qquad \text{Regroup factors.}$$
$$= -6 \cdot x^3 \cdot x \qquad \text{Multiply coefficients.}$$
$$= -6x^4 \qquad \text{Product of powers property}$$

Your turn now **Simplify the expression.**

1. $4a(a^2)$ **2.** $(-2m)(7m^2)$ **3.** $(-x)(8x^2)$ **4.** $(y^5)(5y)$

You can use the distributive property and the properties of exponents to find the product of a monomial and a binomial.

EXAMPLE 2 Using the Distributive Property

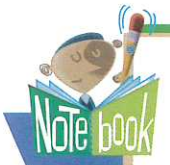
Simplify the expression $2n(4n^2 - 5)$.

$$2n(4n^2 - 5) = (2n)(4n^2) - (2n)(5) \qquad \text{Distributive property}$$
$$= 8n^3 - 10n \qquad \text{Product of powers property}$$

Your turn now **Simplify the expression.**

5. $p(2p + 3)$ **6.** $-t^2(-2t + 8)$ **7.** $n^2(5n^2 - 3)$ **8.** $2x(3x - 4)$

In the activity on page 666, you found powers of products. You can use the rule below to simplify a power of a product.



Power of a Product Property

Words To simplify a power of a product, find the power of each factor and multiply.

Algebra $(ab)^m = a^m \cdot b^m$ **Numbers** $(5 \cdot 2)^3 = 5^3 \cdot 2^3$

EXAMPLE 3 Simplifying a Power of a Product

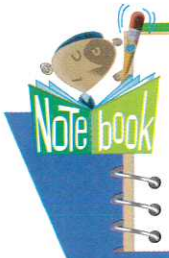
Container The radius of a container is twice its height. Write an expression for the volume of the container. Use the formula $V = \pi r^2 h$.

Solution

The radius is twice the height, so $r = 2h$.

$$V = \pi(2h)^2 h \qquad \text{Substitute } 2h \text{ for } r.$$
$$= \pi(2^2 \cdot h^2)h \qquad \text{Power of a product property}$$
$$= \pi \cdot 4 \cdot h^2 \cdot h \qquad \text{Evaluate the power.}$$
$$= 4\pi h^3 \qquad \text{Product of powers property.}$$

ANSWER An expression for the volume of the container is $V = 4\pi h^3$.



Power of a Power Property

Words To simplify a power of a power, multiply exponents.

Algebra $(a^m)^n = a^{mn}$ **Numbers** $(5^3)^2 = 5^{3 \cdot 2} = 5^6$

EXAMPLE 4 Simplifying a Power of a Power

Simplify the expression $(2y^2)^3$.

$(2y^2)^3 = 2^3 \cdot (y^2)^3$ Power of a product property

$= 8 \cdot y^{2 \cdot 3}$ Power of a power property

$= 8y^6$ Simplify.

Your turn now **Simplify the expression.**

9. $(2^4)^2$ **10.** $(x^6)^2$ **11.** $(5m^3)^2$ **12.** $(a^2b)^2$

13.3 Exercises

More Practice, p. 739

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Getting Ready to Practice

Vocabulary **Match the expression with the rule used to simplify it.**

1. $(2y)^5$

2. $(x^2)^7$

3. $3 \cdot x^4 \cdot x^6$

A. power of a power property

B. power of a product property

C. product of powers property

Simplify the expression.

4. $(5x)(7x^6)$ **5.** $2x(x^2 - 1)$ **6.** $(4y)^3$ **7.** $(z^4)^4$

8. Guided Problem Solving Simplify the expression $\left(\frac{x}{y}\right)^4$.

(1) Write the expression in expanded form.

(2) Simplify by multiplying numerators and multiplying denominators.

(3) Write a rule you could use to find the power of a quotient.

Practice and Problem Solving

HELP with Homework

Example	Exercises
1	9–14
2	15–20
3	22–29
4	30–37

Online Resources CLASSZONE.COM
- More Examples
- eTutorial Plus

Simplify the expression by multiplying the monomials.

9. $(-4x)(5x^3)$ **10.** $(-16t)(-3t^9)$ **11.** $(-x^2)(-3x)$

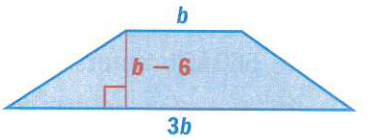
12. $(3s)(-2s^3)$ **13.** $(-b^3)(-b^8)$ **14.** $(-y^2)(y^3)$

Simplify the expression by using the distributive property.

15. $m(m + 4)$ **16.** $2w(3w + 1)$ **17.** $-t(t^2 - 4)$

18. $-8x(x^5 + x)$ **19.** $w^2(-2w - 1)$ **20.** $3k^2(12 - k^5)$

21. Seat Cushion You need fabric for a window seat cushion. Use the trapezoid pattern shown to write a polynomial expression for the area of the top of the cushion. Simplify the expression.



Simplify the expression by using the power of a product property.

22. $(5z)^3$ **23.** $(xyz)^5$ **24.** $(2ab)^4$ **25.** $(-6z)^3$

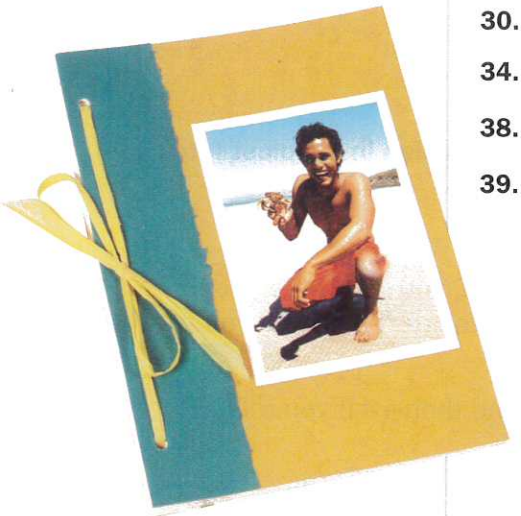
26. $(-dt)^4$ **27.** $(3rs)^2$ **28.** $(-3xy)^3$ **29.** $(10bh)^5$

Simplify the expression by using the power of a power property.

30. $(t^4)^2$ **31.** $(y^2)^2$ **32.** $(c^2)^9$ **33.** $(x^2)^{10}$

34. $(ab^3)^2$ **35.** $(x^2y^2)^3$ **36.** $(3a^2)^2$ **37.** $(2r^3)^3$

38. Compare and Contrast Explain why $(4y)^2$ is different from $4y^2$.

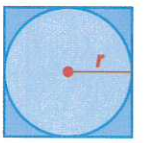


39. Photo Albums You are making photo albums in different sizes. Each page is twice as long as it is wide and needs a 2 inch margin for binding. Write a polynomial expression for the total area of one page.

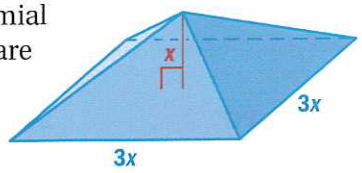
Simplify the expression.

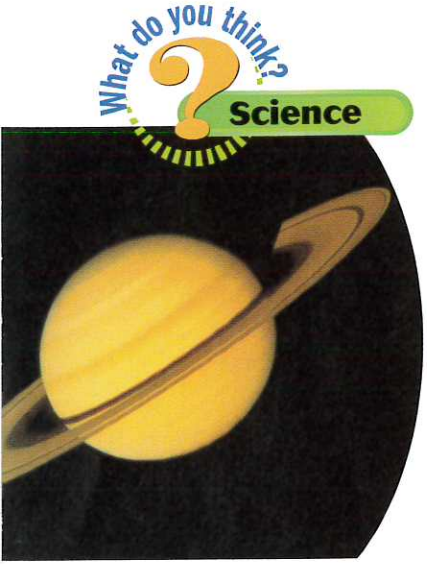
40. $2(5mn^4)^3$ **41.** $-3a^{10}(a^4b^2c)^4$ **42.** $(-2x^4)^3(x^4yz^8)$

43. Critical Thinking Write a ratio comparing the area of the circle to the area of the square. Simplify the ratio. Leave your answer in terms of π.



44. Volume Write and simplify a polynomial expression for the volume of the square pyramid.





Saturn

The moon Pan orbits within a gap in Saturn's rings. Pan's radius is about 10 kilometers. Compare the volumes of Pan and Saturn.

Scientific Notation **Simplify the expression and write it in scientific notation.**

45. $(3 \times 10^4)^3$ **46.** $(9 \times 10^{10})^3$ **47.** $(5 \times 10^7)^4$

Extended Problem Solving **In Exercises 48–50, use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere to find the volumes of spherical objects in our solar system.**

48. Saturn The radius of Saturn is about 6.0×10^4 kilometers.

49. Moon of Saturn The radius of Saturn's moon Dione is about 560 kilometers.

50. Estimate Write a ratio comparing the volume of Saturn to the volume of Dione. Use this ratio to estimate how many times larger Saturn is than Dione.

Challenge **Simplify the expression.**

51. $3[(a^4b^3)^4 \cdot a^8b]^3$ **52.** $\left(\frac{2x^2}{x}\right)^3$ **53.** $\frac{(-2xy)^2}{(x^2)^3}$

Mixed Review

In Exercises 54–57, write the percent as a fraction in simplest form. *(Lesson 7.4)*

54. 55% **55.** 71% **56.** 29% **57.** 18%

58. In one out of every eight holes of mini-golf, you get a hole in one. Find the odds of getting a hole in one on the next hole that you play. *(Lesson 12.7)*

59. Find the probability of rolling first a 2 and then a 4 if you roll a number cube twice. *(Lesson 12.8)*

Test-Taking Practice

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60. Multiple Choice Simplify the expression $4a^2(3a + 1)$.

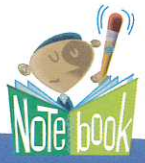
A. $7a^3 + 1$ **B.** $12a^3 + 4a$ **C.** $7a^3 + 4a^2$ **D.** $12a^3 + 4a^2$

61. Multiple Choice Simplify the expression $(-2b^4)^3$.

F. $-8b^{12}$ **G.** $8b^7$ **H.** $8b^{12}$ **I.** $-8b^7$

LESSONS 13.1 TO 13.3

Notebook Review



Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

polynomial, p. 657
binomial, p. 657
trinomial, p. 657
standard form, p. 657

Use Your Vocabulary

Copy and complete the statement.

1. A polynomial with one term is called a ? .
2. A ? is a monomial or a sum of monomials.

13.1 Can you simplify polynomials?

Review

EXAMPLE Simplify the polynomial $4x^2 - 5(x^2 - x + 3 - 2x)$.

$4x^2 - 5(x^2 - x + 3 - 2x)$

$= 4x^2 - 5x^2 + 5x - 15 + 10x$ **Distributive property**

$= -x^2 + 15x - 15$ **Combine like terms.**

Simplify the polynomial and write it in standard form.

3. $10 - 3a^2 + 4a^2 + 8$
4. $6 + z^2 - 3z + z^2 - 5$

13.2 Can you add and subtract polynomials?

Review

EXAMPLE Find the sum $(3x^2 - 2x + 7) + (5x - 9)$.

$3x^2 - 2x + 7$ **Write the second polynomial under the first.**

$+ \quad 5x - 9$ **Arrange like terms in columns.**

$3x^2 + 3x - 2$ **Combine like terms.**

Find the sum or difference.

5. $(n^3 + 4n^2 - 9) + (n^3 + n^2 - 2n + 6)$
6. $(2x^2 + 3x - 1) - (7x^2 - x - 5)$

13.3 Can you multiply monomials?

Review

EXAMPLE Simplify the expression $(x^3)(3x)^2$.

$$(x^3)(3x)^2 = x^3(3^2 \cdot x^2) \quad \text{Power of a product property}$$
$$= x^3 \cdot 9 \cdot x^2 \quad \text{Evaluate the power.}$$
$$= 9x^5 \quad \text{Product of powers property}$$

Simplify the expression.

7. $(8x^8)(6x^2)$ **8.** $(6n^3m)^2$ **9.** $(2ab)^4$ **10.** $(4r^2)(r - 5)$

Stop and Think about Lessons 13.1–13.3

11. Writing Simplify the expression $(2x)(x^2y)$ and explain your steps.

12. Critical Thinking The radius of a cylinder is three times its height. Write a polynomial expression for the surface area of the cylinder using only one variable. Use the formula $S = 2\pi r^2 + 2\pi rh$.

Review Quiz 1

Simplify the polynomial and write it in standard form.

1. $5x^2 + 4x - 3x^2 - 11$ **2.** $-9y^2 + 7y - 2y + 10 - y$

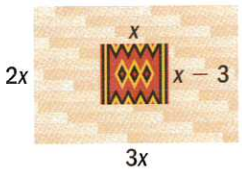
3. $-5k^3 + 2(3k^3 + k - 4)$ **4.** $8r^3 - 4r - 5r^3 - 3r + 1$

Find the sum or difference.

5. $(6n^3 - 2n^2) + (n^3 + 7n^2 - 4n)$ **6.** $(4b^2 - 3b + 8) - (2b^2 - 6)$

7. $(x^2 + 6x + 1) - (2x^2 - 8x + 4)$ **8.** $(3m^2 + m - 9) + (7m^2 + 2)$

9. Area Write a polynomial expression for the area of the floor surrounding the rug in the diagram. Simplify the polynomial.



Simplify the expression.

10. $(x^3)(-5x)$ **11.** $(3t^4)(4t^2)$ **12.** $(2c^3)^4$

13. $(-2y)^4$ **14.** $r^3(3r - 4)$ **15.** $-5d(3d^2 + 2)$

13.4 Hands-on Activity

GOAL
Multiply binomials using algebra tiles.

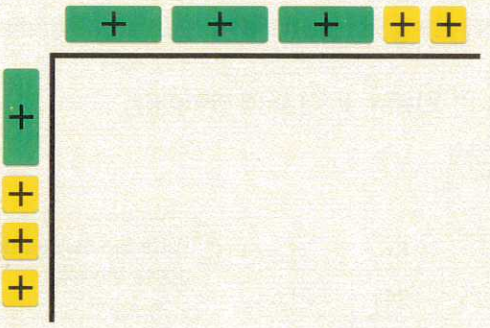
MATERIALS
- algebra tiles

Multiplying Binomials

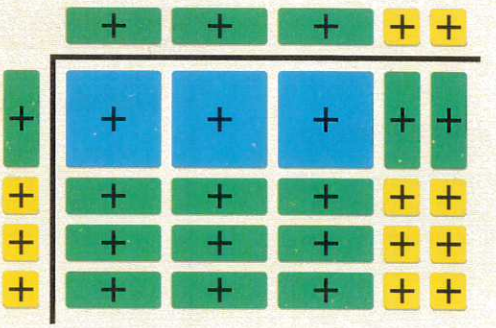
You can model binomial multiplication with algebra tiles.

Explore **Model the product $(x + 3)(3x + 2)$ with algebra tiles.**

1. Model each binomial with algebra tiles. Arrange the first binomial vertically and the second binomial horizontally, as shown.



2. The binomials define a rectangular area with length $(3x + 2)$ units and width $(x + 3)$ units. Fill in the region with the appropriate tiles.



3. The rectangle on the inside of the model represents $3x^2 + 11x + 6$. This is the product of the binomials.

Your turn now **Find the product with algebra tiles. Draw your model.**

1. $(x + 1)(x + 2)$ **2.** $(x + 4)(x + 4)$ **3.** $(x + 2)(2x + 2)$

Stop and Think

Model the expression with algebra tiles. Arrange the tiles in a rectangle. Find the two binomials that have this product.

4. $x^2 + 5x + 6$ **5.** $x^2 + 2x + 1$ **6.** $3x^2 + 8x + 4$

LESSON 13.4

Multiplying Binomials

BEFORE You multiplied monomials and polynomials.

NOW You'll multiply binomials.

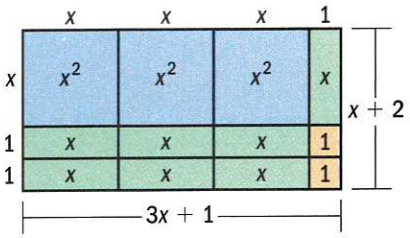
WHY? So you can find an account balance after 2 years, as in Ex. 22.

Word Watch

Review words

polynomial, p. 657
binomial, p. 657

You can use a visual model to multiply binomials. The model below shows that the product $(x + 2)(3x + 1)$ equals $3x^2 + 7x + 2$.



You can multiply two binomials using a table or a vertical method.

HELP with Notetaking

You should summarize key ideas about polynomials in your notebook.

EXAMPLE 1 Multiplying Binomials with a Table

Find the product $(-3x + 2)(8x + 7)$ and simplify.

Write the first polynomial on the left of the table.

Write the second polynomial above the table.

	$8x$	7
$-3x$	$-24x^2$	$-21x$
2	$16x$	14

Multiply to fill in the table.

The product is $-24x^2 - 21x + 16x + 14$. Combine like terms.

ANSWER The product is $-24x^2 - 5x + 14$.

EXAMPLE 2 Multiplying Binomials Vertically

Find the product $(2x - 5)(3x + 4)$ and simplify.

$$\begin{array}{rl} & 2x - 5 \\ \times & 3x + 4 \\ \hline & 8x - 20 \\ 6x^2 - 15x & \\ \hline 6x^2 - 7x - 20 & \end{array}$$

Write the first binomial.

Write the second binomial.

Multiply $4(2x - 5)$.

Multiply $3x(2x - 5)$. Line up like terms.

Add $8x - 20$ and $6x^2 - 15x$.

HELP with Vocabulary

Compound interest is earned on the original amount of money in an account and on the interest already earned.

EXAMPLE 3 Multiplying Binomials Horizontally

Banking You deposit \$1 into a savings account with interest compounded annually. The balance of the account after two years can be found using the expression $(1 + r)^2$, where r represents the interest rate. Expand this expression and simplify.

Solution

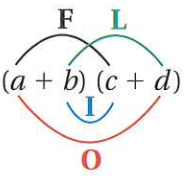
To expand the expression, multiply 2 binomials.

$$(1 + r)^2 = (1 + r)(1 + r) \qquad (1 + r)^2 \text{ means } (1 + r)(1 + r).$$
$$= 1(1 + r) + r(1 + r) \qquad \text{Distributive property}$$
$$= 1 + r + r + r^2 \qquad \text{Distributive property}$$
$$= 1 + 2r + r^2 \qquad \text{Combine like terms.}$$
$$= r^2 + 2r + 1 \qquad \text{Write in standard form.}$$

Your turn now **Find the product and simplify.**

1. $(x + 1)(x + 3)$ **2.** $(b - 4)(b - 3)$ **3.** $(3t - 4)(t + 2)$

The FOIL Method The letters in the word FOIL can help you remember how to multiply binomials. The letters should remind you of the words First, Outer, Inner, and Last.



EXAMPLE 4 Multiplying with the FOIL Method

Find the product $(2x + 3)(3x - 1)$ and simplify.

F		O		I		L	
First	+	Outer	+	Inner	+	Last	
$2x \cdot 3x$	+	$2x \cdot (-1)$	+	$3 \cdot 3x$	+	$3 \cdot (-1)$	Group terms.
$6x^2$	+	$(-2x)$	+	$9x$	+	(-3)	Multiply.
		$6x^2 + 7x - 3$					Combine like terms.

Your turn now **Find the product and simplify.**

4. $(d + 6)(d + 5)$ **5.** $(x - 3)(x - 1)$ **6.** $(5s + 3)(2s - 4)$

13.4 Exercises

More Practice, p. 739

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Getting Ready to Practice

HELP with Homework

Example	Exercises
1	9–17
2	9–17
3	22
4	9–17

Online Resources CLASSZONE.COM

- More Examples
- eTutorial Plus

1. Vocabulary Copy and complete: A polynomial with two terms is called a ?.

Find the product and simplify.

2. $3x(x - 4)$ **3.** $2m(3m + 1)$ **4.** $-2y(y + 5)$

5. $(y - 4)(y + 1)$ **6.** $(g + 3)(g + 7)$ **7.** $(z + 4)(z - 2)$

8. Find the Error Describe and correct the error in the solution.

$$(x + 2)(x - 4)$$
$$= x \cdot x + x \cdot 4 + 2 \cdot x + 2 \cdot (-4)$$
$$= x^2 + 4x + 2x - 8$$
$$= x^2 + 6x - 8$$

Practice and Problem Solving

Find the product and simplify.

9. $(x + 9)(x - 2)$ **10.** $(p + 6)(p + 4)$ **11.** $(a + 10)(a - 4)$

12. $(2m + 3)(m - 7)$ **13.** $(3q - 1)(q - 1)$ **14.** $(b - 3)(9b + 4)$

15. $(6r + 7)(r - 1)$ **16.** $(t - 1)(-3t - 4)$ **17.** $(-x - 5)(11x - 12)$

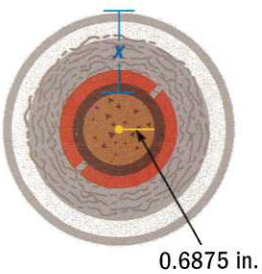
Critical Thinking **Find the product and simplify.**

18. $(x + 3)(x - 3)$ **19.** $(x - 4)(x + 4)$ **20.** $(x + 1)(x - 1)$

21. Look for a Pattern Describe the pattern in the binomials and their products in Exercises 18–20.

22. Savings Account You deposit \$50 into a savings account with interest compounded annually. The expression $50(1 + r)^2$, where r is the interest rate, gives the account balance after 2 years. Expand this expression and simplify. Find the account balance for $r = 0.03$.

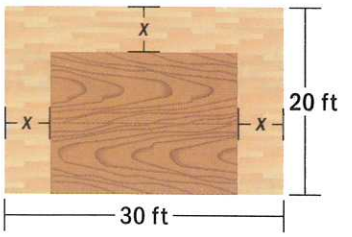
23. Baseball The middle of a baseball is a cork sphere with a radius of 0.6875 inch. Use the formula $S = 4\pi r^2$ to write a polynomial expression for the surface area of the baseball. Expand the expression and simplify.



Find the product and simplify.

24. $\left(\frac{1}{2}x + 2\right)(4x - 6)$ **25.** $(9b - 12)\left(\frac{1}{3}b - 6\right)$ **26.** $(n^2 - 2)(n^2 + 1)$

27. Stage Design You are building a platform on stage for a school talent show. Write and simplify a polynomial expression for the area of the platform using the design shown. Then find the area when x is 5 feet.



28. Writing Explain why $(x + 3)^2$ does not equal $x^2 + 9$.

29. Challenge Find the product and simplify: $(x + 4)(3x^2 - 2x + 1)$.

30. Savings You put \$20 into a savings account with interest compounded annually. The expression $20(1 + r)^3$, where r is the interest rate, gives the account balance after 3 years. Expand this expression and simplify. Find the account balance for $r = 0.05$.

31. Work Backward Find the unknown binomial in the equation $x^2 + 8x + 7 = (\ \_?\_\ )(x + 7)$.

Mixed Review

Graph the linear equation. *(Lesson 11.4)*

32. $y = 6x - 4$ **33.** $y = x - 3$ **34.** $y = -2x + 7$

Find the product. *(Lesson 13.3)*

35. $-4r(r + 6)$ **36.** $3c(4c^2 + 2c)$ **37.** $-5x(-3x + 2)$

Basic Skills **Plot the point in a coordinate plane.**

38. $(2, -9)$ **39.** $(-7, 6)$ **40.** $(-3, -8)$ **41.** $(0, 4)$

Test-Taking Practice

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42. Multiple Choice Find the product $(x + 6)(x - 2)$.

A. $x^2 - 4x - 12$ **B.** $x^2 - 4x + 4$

C. $x^2 + 4x + 12$ **D.** $x^2 + 4x - 12$

43. Multiple Choice Find the product $(2x + 1)(x - 5)$.

F. $2x^2 - 9x - 5$ **G.** $x^2 - 9x - 5$

H. $2x^2 + 11x - 5$ **I.** $x^2 + 11x - 5$

13.5 Problem Solving Strategies

- Look for a Pattern
- Draw a Diagram
- Act It Out
- Work Backward
- Draw a Graph
- Break into Parts

Draw a Graph

Daylight The table below shows the total hours of daylight, to the nearest quarter hour, in Anchorage, Alaska, on the 20th day of each month. The table is missing data for the month of July. Estimate the total hours of daylight on July 20.

Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Hours of Daylight	$6\frac{3}{4}$	$9\frac{1}{2}$	$12\frac{1}{4}$	$15\frac{1}{4}$	$17\frac{3}{4}$	$19\frac{1}{4}$	?	$15\frac{1}{2}$	$12\frac{1}{2}$	$9\frac{3}{4}$	7	$5\frac{1}{2}$

1 Read and Understand

Read the problem carefully.

You need to estimate the total hours of daylight on July 20.

2 Make a Plan

Decide on a strategy to use.

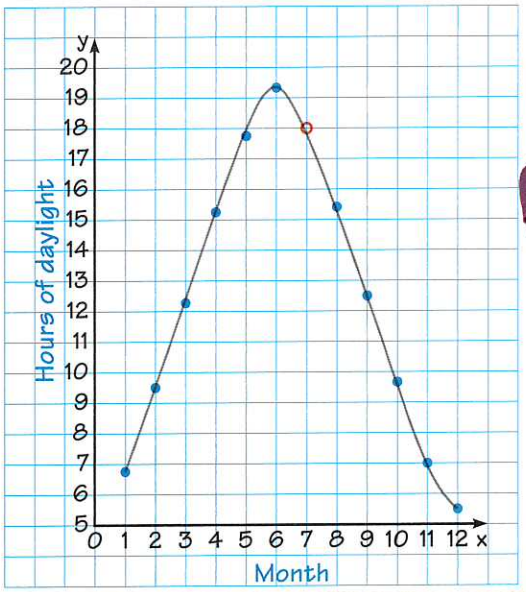
You can estimate the hours by drawing a graph. Use the table of values to sketch a curve.

3 Solve the Problem

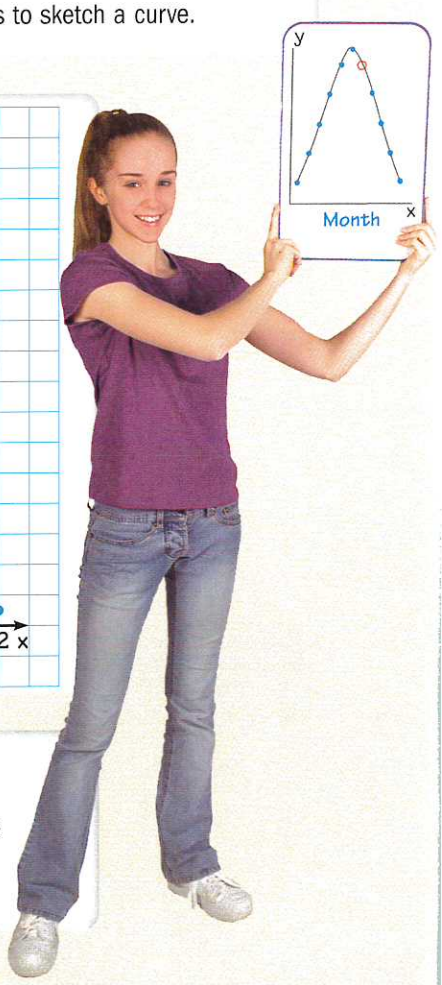
Reread the problem and draw a graph.

The data in the table are ordered pairs. Let the x-axis show the date (Jan. 20 = 1, Feb. 20 = 2, etc.). Let the y-axis show the hours of daylight. Plot the points in the coordinate plane.

Connect the points with a curve, and use it to estimate the total hours of daylight on July 20.



ANSWER There are about 18 hours of daylight on July 20.



4 Look Back

Because the graph decreases from June to December it is reasonable to say that there are between $19\frac{1}{4}$ and $15\frac{1}{2}$ hours of daylight on July 20. A reasonable estimate is 18 hours.

Practice the Strategy

Use the strategy *draw a graph*.

1. **E-mail** You e-mail two copies of a joke on January 1. Each recipient e-mails two copies of the joke on the day after receiving it. How many e-mails are sent on January 7?

2. **Square Root** Copy and complete the table.

x	1	4	9	16	25	36
$\sqrt{x}$	?	?	?	?	?	?

Estimate the square roots below.

a. $\sqrt{7}$ **b.** $\sqrt{30}$ **c.** $\sqrt{20}$

3. **Cutting** The area of a poster board is 1000 square inches. You repeatedly cut the poster in half, setting aside one half each time. Estimate how many cuts you will make before the area of the remaining piece is less than 1 square inch.

4. **Kiwi Bird** A female kiwi bird weighs from 3 to 9 pounds. A kiwi bird's egg weighs about $\frac{1}{5}$ of the mother's body weight. Estimate the weight of a female bird whose egg weighs less than 0.7 pound.

5. **Tutoring** For each hour you spend tutoring after school, you earn 5 extra-credit points. Make a table that shows how many extra-credit points you can earn for hours you spend tutoring. Estimate how many extra-credit points you would earn after tutoring for 1 hour and 45 minutes.

Mixed Problem Solving

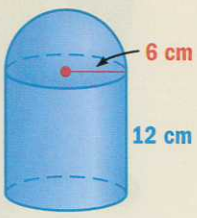
Use any strategy to solve the problem.

6. **Peanut Butter** In a poll, 44% of the people said the best thing to eat with peanut butter is jelly. About 23% chose chocolate. Only 15% preferred marshmallow spread with peanut butter. The rest of the people chose bananas. If 2916 people participated in the poll, about how many people choose bananas with peanut butter?

7. **Draw** Copy the grid. How many squares can you draw by connecting dots?

8. **Encyclopedia** You have a two-volume music encyclopedia. Each volume has 600 pages, and its front and back covers are each 0.4 centimeter thick. If the books sit in the usual order on a shelf, what is the distance, in centimeters, from the first page of Volume 1 to the last page of Volume 2?

9. **Solid** Find the volume of the solid shown. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Round to the nearest hundredth of a cubic centimeter.



LESSON 13.5

Non-Linear Functions

BEFORE You wrote function rules and graphed linear functions.

NOW You'll use function notation and graph non-linear functions.

WHY? So you can find the height of a falling penny, as in Ex. 11.

In the Real World

Word Watch

function notation, p. 680
vertical line test, p. 681

Juggling You are juggling three balls. The height of one ball, in feet, is found with the equation $h = -16t^2 + 20t + 3$, where t is seconds that pass after you let go of the ball. Write the equation in function notation. Use this function to find the height of one ball 0.5 second after you let go of it.

In Lesson 11.1, you wrote functions as equations in x and y. You used x to name the input and y to name the output. Sometimes it is useful to use **function notation** instead.

equation in x and y	function notation
$y = 3x - 4$	$f(x) = 3x - 4$

The symbol $f(x)$ is read as "the function of f at x" or "f of x."

EXAMPLE 1 Using Function Notation

Write a function that models the height of a ball x seconds after you let go of it. Use function notation. Evaluate for $x = 0.5$.

Solution

Let $f(x)$ = height in feet and x = time in seconds.

$f(x) = -16x^2 + 20x + 3$ — Write the height equation above in function notation.

$f(0.5) = -16(0.5)^2 + 20(0.5) + 3$ — Substitute 0.5 for x.

$f(0.5) = -16(0.25) + 10 + 3$ — Evaluate.

$f(0.5) = 9$

ANSWER The function is $f(x) = -16x^2 + 20x + 3$. The height of the ball is 9 feet after 0.5 second.

Your turn now **Rewrite using function notation.**

1. $y = x^2$
2. $y = 3x^2 + 4$
3. $y = -\frac{1}{2}x^2$

Graphing Functions The function in Example 1 is non-linear. A non-linear function has a graph that is not a straight line. You can graph non-linear functions by first making a table of values.

HELP with Reading

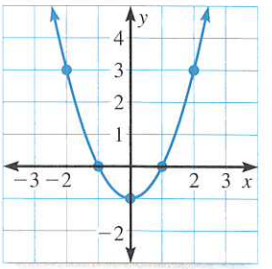
$f(x)$ does not mean "f times x." It means "the value of the function at x."

EXAMPLE 2 Graphing a Non-Linear Function

Graph the function $f(x) = x^2 - 1$.

(1) Choose several x-values and make a table of values.

x	-2	-1	0	1	2
$f(x)$	3	0	-1	0	3



(2) List the solutions as ordered pairs.
$(-2, 3), (-1, 0), (0, -1), (1, 0), (2, 3)$

(3) Plot the ordered pairs. Then draw a smooth curve through the points, as shown.

Your turn now **Graph the function using a table of values.**

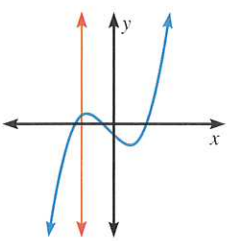
4. $f(x) = -x^2 + 4$ **5.** $f(x) = x^2 + 1$ **6.** $f(x) = 2x^2$

Vertical Line Test You can use the **vertical line test** to tell whether a graph represents a function. If a vertical line intersects the graph at more than one point, then the graph does *not* represent a function. Remember, a function has exactly one output value for each input value.

EXAMPLE 3 Using the Vertical Line Test

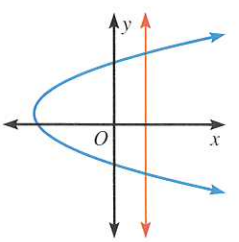
Tell whether the graph represents a function.

a.



No vertical line intersects the graph at more than one point. So, the graph represents a function.

b.



A vertical line intersects the graph at more than one point. So, the graph does *not* represent a function.

13.5 Exercises

More Practice, p. 739

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HELP with Homework

Example	Exercises
1	8-11
2	12-17
3	18-20

Online Resources
CLASSZONE.COM
- More Examples
- eTutorial Plus

Getting Ready to Practice

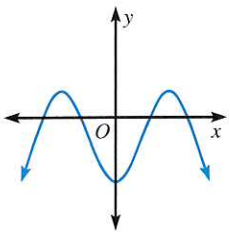
1. **Vocabulary** Which is written in function notation?

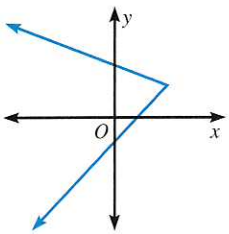
   A. $f = 2x + 4$ **B.** $f(x) = 2x + 4$ **C.** $2(f) + 4$

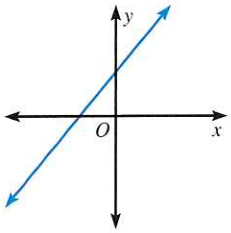
Evaluate the function for $x = -2$, 0, and 2.

2. $f(x) = x^2$ 3. $f(x) = x^2 - 5$ 4. $f(x) = -3x^2$

Tell whether the graph represents a function.

5. 

6. 

7. 

Practice and Problem Solving

Rewrite using function notation.

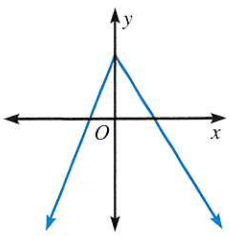
8. $y = 4x^2$ 9. $y = 2x^2 - x$ 10. $y = -x^2 + 10$

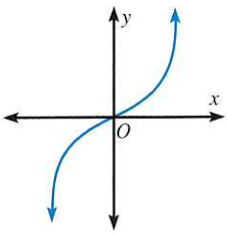
11. **Penny Drop** You drop a penny into a well and hear it hit the water after 3 seconds. The function $d = -16t^2 + 4$ gives the elevation of the penny, in feet, after it falls for t seconds. Write this in function notation, and find how far the penny falls before hitting the water.

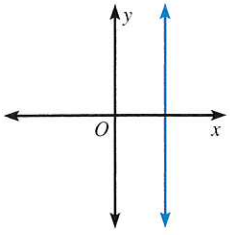
Graph the function using a table of values with $x = -3, -2, -1, 0, 1, 2,$ and 3.

12. $f(x) = 4x^2$ 13. $f(x) = x^2 + 8$ 14. $f(x) = -x^2 + 5$
15. $f(x) = -x^2$ 16. $f(x) = 3x^2 - 4$ 17. $f(x) = -2x^2 - 1$

Tell whether the graph represents a function.

18. 

19. 

20. 

21. Reflecting Pool A rectangular reflecting pool is 50 feet long and 30 feet wide, but you do not know how deep the water is. Write a function that you can use to find the volume of water in the pool for different depths.

22. Baseball A ball is hit 75 mi/h, or 110 ft/sec, into center field. The height of the ball, in feet, is found with the function $f(x) = -16x^2 + 110x + 3$, where x is the number of seconds after the ball is hit. Graph this function. Use the graph to estimate how many seconds will pass before the ball lands on the ground.

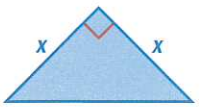
Graph the function using a table of values.

23. $f(x) = -\frac{1}{2}x^2$ **24.** $f(x) = 5 - 5x^2$ **25.** $f(x) = 4x^2 + x$

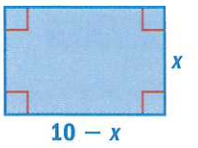
26. $f(x) = x^2 - 3x$ **27.** $f(x) = (x - 1)^2$ **28.** $f(x) = (x + 2)^2$

29. Make a Table Graph the function $f(x) = x^3$ using a table of values. Describe how the graph of this function is different from the graph of the function $f(x) = x^2$. Be sure to include negative and positive x-values in your table.

30. Draw a Graph Write a function for the area of the triangle. Graph the function using a table of values. Estimate the value of x if the area of the triangle is 30 square inches.



31. Interpret Write a function for the area of the rectangle shown. Graph the function using a table of values, and use it to find the greatest possible area of the rectangle. If the rectangle has the maximum area, what are its length and width?



Work Backward **Write a function in function notation for the values in the table.**

32.

x	−2	−1	0	1	2
$f(x)$	5	2	1	2	5

33.

x	−4	−2	0	2	4
$f(x)$	−64	−8	0	8	64

34. Challenge You deposit \$20 into a savings account that earns 2% interest compounded monthly. The expression $20(1.02)^t$ gives the account balance after t months. Write a function for the account balance, and find the balance after 1, 2, and 3 months.

35. Population In 2001, the United States population was about 2.8×10^8 people and growing 0.8% each year. You can predict the future population with the expression $(2.8 \times 10^8)(1.008)^t$, where t is the number of years after 2001. Write the expression as a function and use it to predict the population of the United States in 2005.

Mixed Review

In Exercises 36–43, solve the equation. *(Lesson 3.3)*

36. $3x + 1 = 7$ **37.** $2x - 3 = 5$ **38.** $-x + 1 = 2$ **39.** $6x - 1 = -11$

40. $2x + 5 = -17$ **41.** $2x + 4 = 10$ **42.** $-3x - 4 = 11$ **43.** $5x + 3 = -12$

44. List four solutions of the equation $2x + 4y = -12$. *(Lesson 11.3)*

Find the product and simplify. *(Lesson 13.4)*

45. $(x + 2)(x + 2)$ **46.** $(3z - 2)(2z - 1)$ **47.** $(5a - 1)(a + 3)$

Choose a Strategy **Use a strategy from the list to solve the following problem. Explain your choice of strategy.**

Problem Solving Strategies
- Draw a Diagram
- Break into Parts
- Solve a Simpler Problem

48. You are saving pennies in a coffee can. On the first day, you put one penny in the can. On the second day, you put two pennies in the can. On the third day, you put three pennies in the can. If you continue this method of saving, how many pennies will be in the can on day 100?

Test-Taking Practice

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49. **Extended Response** Graph the four functions below using tables of values. Describe the differences in the graphs. Explain the effect of a negative coefficient on a graph.

$f(x) = x^2$ $f(x) = -x^2$ $f(x) = \frac{1}{2}x$ $f(x) = -\frac{1}{2}x$

BRAIN GAME

Number Crunch

Why was ten afraid of seven?

Use the function $f(x) = 2x^2 - 3x + 7$ to break the code.

?	?	?	?	?	?	?	?	?	?	?	?
−3	5	0	3	−4	1	5	1	5	2	5	6

?	?	?	?	?	?	?	?	?
5	4	−2	−1	−5	6	4	6	5

A	B	C	E	G	H	I	N	S	T	U	V
16	34	7	42	21	12	27	61	6	72	51	9

13.5 Technology Activity

GRAPHING CALCULATOR

Graphing Non-Linear Functions

GOAL Use a graphing calculator to graph non-linear functions.

Example **Use a graphing calculator to compare the functions.**

$y_1 = x^2$ $y_2 = 2x^2$ $y_3 = 3x^2$ $y_4 = 4x^2$

HELP with Technology

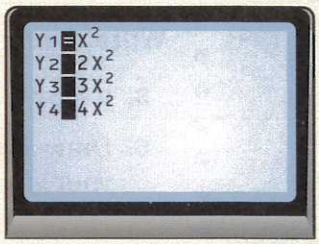
You may need to adjust your viewing window in order to see the graphs.

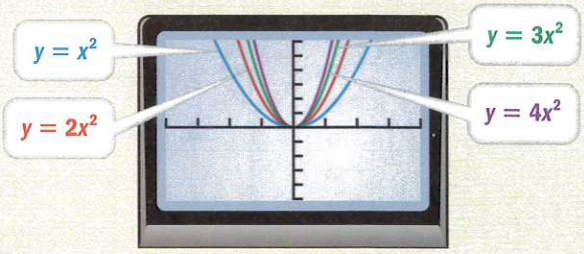
Solution

Use the following keystrokes to enter the functions into a graphing calculator:

	Keystrokes
	Y=
Y_1	x x^2 ENTER
Y_2	2 x x^2 ENTER
Y_3	3 x x^2 ENTER
Y_4	4 x x^2
	GRAPH

Display





ANSWER The graphs are curves that pass through (0, 0). As the coefficient of x^2 increases, the curve gets narrower.

Your turn now **Graph the functions using a graphing calculator. Describe the pattern in the graphs.**

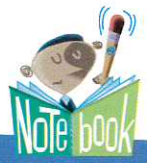
1. $y = x^2 + 5$ **2.** $y = x^2 - 5$ **3.** $y = x^2 + 7$ **4.** $y = x^2 - 7$

Graph the functions. Describe the pattern in the graphs.

5. $y = -x^2$ **6.** $y = -2x^2$ **7.** $y = -3x^2$ **8.** $y = -4x^2$

LESSONS 13.4 TO 13.5

Notebook Review



Review the vocabulary definitions in your notebook.

Copy the review examples in your notebook. Then complete the exercises.

Check Your Definitions

function notation, p. 680

vertical line test, p. 681

Use Your Vocabulary

Copy and complete the statement.

1. The equation $f(x) = 7x - 4$ is written using ?.

2. The ? helps you tell whether a graph represents a function.

Write the function using function notation.

3. $y = 5x - 12$ **4.** $y = 2x^3 + 8$ **5.** $y = x^3 + 3x^2 - 10$

6. Draw a graph that is *not* a function. Use the vertical line test to show why your graph is *not* a function.

13.4 Can you multiply binomials?

Review

EXAMPLE Find the product and simplify.

a. $(4x - 7)(2x + 3)$

$$\begin{array}{rl} & 4x - 7 \\ \times & 2x + 3 \\ \hline & 12x - 21 \\ 8x^2 - 14x & \\ \hline 8x^2 - 2x - 21 & \end{array}$$

Write the first binomial.

Write the second binomial.

Multiply $3(4x - 7)$.

Multiply $2x(4x - 7)$. Line up like terms.

Add $12x - 21$ and $8x^2 - 14x$.

b. $(x + 3)(x + 2)$

$= x(x + 2) + 3(x + 2)$ Distributive property

$= x^2 + 2x + 3x + 6$ Distributive property

$= x^2 + 5x + 6$ Combine like terms.

Find the product and simplify.

7. $(x + 5)(x + 7)$ **8.** $(g + 10)(g - 2)$ **9.** $(y - 4)(3y - 1)$

13.5 Can you graph non-linear functions?

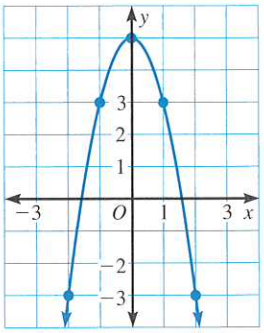
Review

EXAMPLE Graph $f(x) = -2x^2 + 5$.

Choose several x-values and make a table of values.

x	-2	-1	0	1	2
$f(x)$	-3	3	5	3	-3

List the solutions as ordered pairs.
$(-2, -3)$, $(-1, 3)$, $(0, 5)$, $(1, 3)$, $(2, -3)$



Graph the ordered pairs. Draw a smooth curve through them, as shown.

Graph the function using a table of values.

10. $f(x) = 3x^2$ **11.** $f(x) = -x^2 + 3$ **12.** $f(x) = 2x^2 - 6$

Stop and Think about Lessons 13.4–13.5

13. Writing Explain how you can use the vertical line test to tell whether a graph represents a function.

Review Quiz 2

Find the product and simplify.

1. $(x - 1)(x + 9)$ **2.** $(a + 4)(a + 9)$ **3.** $(m - 2)(m - 8)$

4. $(4y + 5)(y - 2)$ **5.** $(b + 3)(4b - 3)$ **6.** $(3z - 2)(2z - 7)$

Rewrite using function notation.

7. $y = 3x + 9$ **8.** $y = -2x^2 - 4$ **9.** $y = 19 - x + x^2$

Evaluate the function for $x = -2, -1, 0, 1$, and 2.

10. $f(x) = 2 - x^2$ **11.** $f(x) = \frac{1}{2}x^2 - 6$ **12.** $f(x) = x^2 + x$

Graph the function using a table of values.

13. $f(x) = -x^2 + 1$ **14.** $f(x) = \frac{1}{4}x^2$ **15.** $f(x) = -3x^2 - 4$

CHAPTER 13

Chapter Review

Vocabulary

polynomial, p. 657
binomial, p. 657
trinomial, p. 657
standard form, p. 657
function notation, p. 680
vertical line test, p. 681

Vocabulary Review

Copy and complete the statement.

1. The polynomial $x^3 - 2x + 1$ is a ?.
2. A polynomial is written in ? if the exponents of the variable decrease from left to right.
3. You can use the ? to tell whether a graph represents a function.

Classify the polynomial as a *monomial*, a *binomial*, or a *trinomial*.

4. $5x^3 + 2x + 3$
5. $5a^3$
6. $5y + 3$
7. $-r + 3$

Review Questions

Write the polynomial in standard form. *(Lesson 13.1)*

8. $7 - 2a^2 + 10a$
9. $5x - 3x^3 - 4 + x^2$
10. $4 + 7y^2 - 8y^3 + y$
11. $9t + 8 - t^2 + 6t^3$
12. $9 + 2m^5 + m^2 - m^4$
13. $25 + n^2 - 3n^4 + 5n$

Simplify the polynomial and write it in standard form. *(Lesson 13.1)*

14. $8k + 1 + 3k + k^2 - 4$
15. $5w - 2w + 2w^2 - 8$
16. $6p^2 + 9(2p^3 + 3 + p^2)$
17. $3x^2 + 4(7 - x^2 + 4x)$
18. $-8(2s - 3s^2 + 7) + 4s^3$
19. $4(5y - 2y^2 + 11) - 2y^2$

Find the sum or difference. *(Lesson 13.2)*

20. $(10q^2 - 6) + (q^2 + 1)$
21. $(2p^2 + 9p) - (5p^2 + 9p)$
22. $(3x^2 - x + 7) - (6x^2 + 4x - 11)$
23. $(7y^2 + y - 4) + (y^2 - y - 1)$
24. $4(m^2 - 3m) + 5(2m^2 - m)$
25. $-2(v^3 - v^2 + v) - 3(v^3 + 4v^2)$

Review Questions

Simplify the expression. *(Lesson 13.3)*

26. $(3r^2)^3$ **27.** $(2xy)^3$ **28.** $(7z)(-4z)^2$

29. $(-6a^2b^4)^3$ **30.** $(-c^3)(2c^4)$ **31.** $(-9n)(-7p^2)^4$

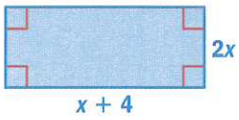
Simplify the expression. *(Lesson 13.3)*

32. $x^2(5x - 7)$ **33.** $-3a(a^2 - 2a)$ **34.** $4y(6y^2 - 8y)$

35. $-y^3(-y^2 + 11y)$ **36.** $7b(14 - 6b^2)$ **37.** $-6g(3g^2 + 10g)$

Write a polynomial expression for the area of the figure and simplify. *(Lesson 13.3)*

38.



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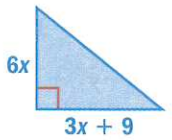


40.

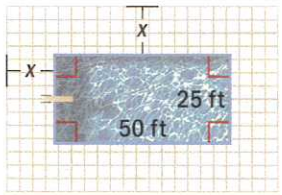
Find the product and simplify. *(Lesson 13.4)*

41. $(t + 3)(t - 4)$ **42.** $(2x + 3)(x + 5)$ **43.** $(q - 7)(q - 9)$

44. $(m - 4)(m - 5)$ **45.** $(3d + 8)(d - 6)$ **46.** $(h + 3)(2h - 5)$

47. $(2k - 9)(-4k - 1)$ **48.** $(2n + 6)(3n - 12)$ **49.** $(-2b + 1)(-b - 4)$

50. Write a polynomial expression for the area of the swimming pool's walkway and simplify. The walkway is x ft wide. *(Lesson 13.4)*



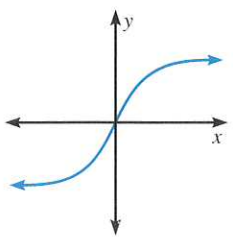
Graph the function using a table of values. *(Lesson 13.5)*

51. $f(x) = x^2 + 3$ **52.** $f(x) = 7 - x^2$

53. $f(x) = x^2 - 4$ **54.** $f(x) = 3x^2 + 1$

Tell whether the graph represents a function. *(Lesson 13.5)*

55.



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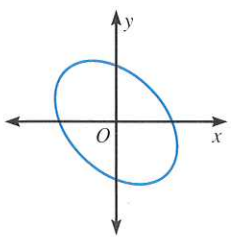

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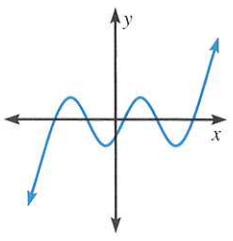


CHAPTER 13

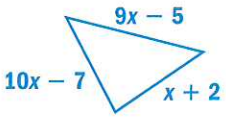
Chapter Test

Simplify the polynomial and write it in standard form.

1. $10x - 7x + 4 + x^2 - 3x + 4$
2. $-y + 6(y^2 - y^3 + 1)$
3. **Apple Picking** An apple falls from a 28 foot tall tree. The height of the apple, in feet, after t seconds of falling, can be found using the polynomial $-16t^2 + 28$. Find the apple's height after 0.5 second.

Find the sum or difference.

4. $(3r^2 + 4r - 7) + (-r^2 - r + 11)$
5. $(4s^2 - 11s) - (s^2 - 6s + 21)$
6. $(4a^5 - a) + (-3a^5 + 1)$
7. $(-y^2 + 12) + (8y^2 - 10)$
8. $(5x^6 - 3x^2 + x) - (4x^2 + 2x)$
9. $(-7z^3 + z^2 - 5) - (z^3 - 3z^2 + 1)$
10. **Perimeter** Write a polynomial expression for the perimeter of the triangle. Simplify the polynomial.



Simplify the expression.

11. $(3a^2)(5a^2b)$
12. $(9z)^2$
13. $(3d^2)^4$
14. $(-2w^4)^3$
15. $(-2p^2)^4$
16. $(x^4y)^8$
17. $(4n^2)(-3n)$
18. $(3r)^3(3r)$

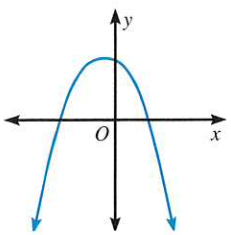
Find the product and simplify.

19. $m^2(3m^2 + 8m)$
20. $7n(n^2 - 2n)$
21. $3p(2p^2 + 3p)$
22. $(2x + 7)(3x + 2)$
23. $(4y + 12)(y - 3)$
24. $(z - 9)(5z + 8)$

Graph the function using a table of values.

25. $f(x) = -5x^2$
26. $f(x) = x^2 + 3$
27. $f(x) = x^2 - 1$

Tell whether the graph represents a function.

28.


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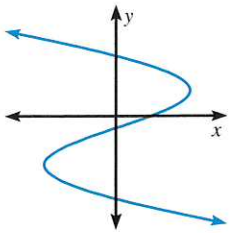

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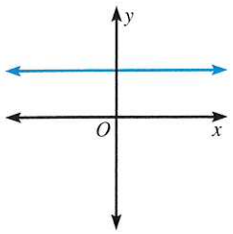


Chapter Standardized Test

Test-Taking Strategy **Think positively during a test. This will help you keep up your confidence and enable you to focus on each question.**

Multiple Choice

1. Simplify the polynomial and write it in standard form.

$10 - 5x^2 + 4x^3 + x^2 - 8$

A. $2 - 4x^2 + 4x^3$ **B.** $4x^3 + 6x^2 + 2$

C. $4x^3 - 4x^2 + 2$ **D.** $2 + 6x^2 + 4x^3$

2. Find the sum.

$(4x^2 + 4x - 5) + (5x^2 - x - 8)$

F. $9x^2 + 3x - 13$ **G.** $9x^2 + 3x - 3$

H. $9x^2 - 3x - 13$ **I.** $9x^2 + 3x + 3$

3. Find the difference.

$(2x^2 - 3x + 2) - (x^2 + 3x + 2)$

A. $x^2 - x$ **B.** $x^2 - 6x$

C. $x^2 + x + 4$ **D.** $x^2 - 6x + 4$

4. Simplify the expression $(2x^2)(x^3y)$.

F. $2x^6y$ **G.** $2xy^5$ **H.** $2x^5y$ **I.** $2x^5y^5$

5. Simplify $11x(x^2 - 2x - 8)$.

A. $11x^3 - 22x^2 + 88$

B. $11x^3 - 22x^2 - 88x$

C. $11x^2 + 22x - 88$

D. $11x^2 + 22x + 88x$

6. Simplify the expression $(5pq)^2$.

F. $10pq$ **G.** $25pq^2$ **H.** $10pq^2$ **I.** $25p^2q^2$

7. Simplify the expression $(-3n^2)^3$.

A. $-27n^6$ **B.** $27n^6$ **C.** $-9n^6$ **D.** $9n^6$

8. Find the product $(x + 3)(x - 4)$ and simplify.

F. $x^2 + 7x - 12$ **G.** $x^2 - x - 12$

H. $x^2 + x - 12$ **I.** $x^2 - x - 7$

9. You deposit \$25 into a savings account with interest compounded annually. The account balance after 2 years is given by the expression $25(r + 1)^2$. Expand this expression and simplify.

A. $25r^2 + 2r + 25$ **B.** $25r^2 + 25r + 1$

C. $25r^2 + 50r + 1$ **D.** $25r^2 + 50r + 25$

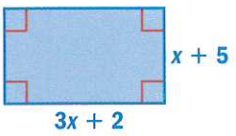
10. Find the value of the function for $x = -2$.

$f(x) = -3x^2 + 5$

F. -7 **G.** 7 **H.** 11 **I.** 17

Short Response

11. Write an expression for the area of the rectangle. Simplify the expression, and write it in standard form.



Extended Response

12. Graph each function using a table of values. Explain how the graphs are different.

$f(x) = x^2$

$f(x) = x^2 + 1$

$f(x) = x^2 + 2$

Describe the graph of the function $f(x) = x^2 + 125$.

Strategies for Answering

Extended Response Questions

Scoring Rubric

Full credit
- answer is correct, *and*
- work or reasoning is included

Partial credit
- answer is correct, but reasoning is incorrect, *or*
- answer is incorrect, but reasoning is correct

No credit
- no answer is given, *or*
- answer makes no sense

Problem

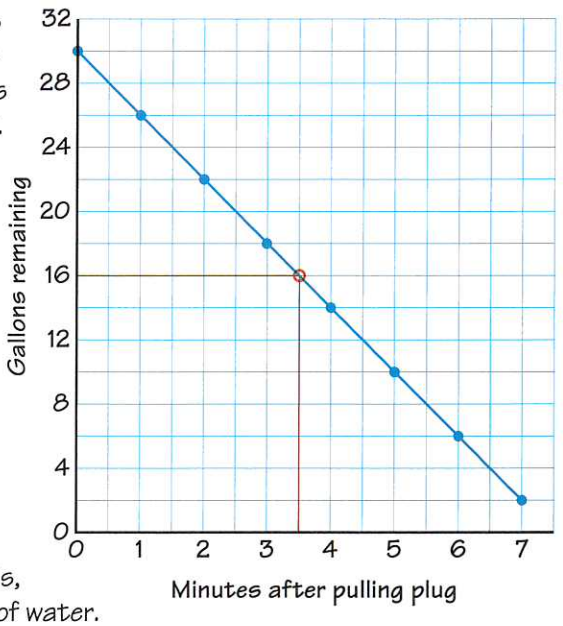
A tank contains 30 gallons of water. You pull out the drain plug, and water begins to flow from the tank at a rate of 4 gallons per minute. Make a table and draw a graph that shows the amount of water in the tank as the water drains. After how much time will the tank contain exactly 16 gallons of water? Give your answer in minutes and seconds. Explain how you found your answer.

Full credit solution

In the graph, the horizontal axis shows minutes after pulling the plug, and the vertical axis shows the gallons of water in the tank.

The table and graph are correct and reflect an understanding of the problem.

minutes	gallons
0	30
1	26
2	22
3	18
4	14
5	10
6	6
7	2



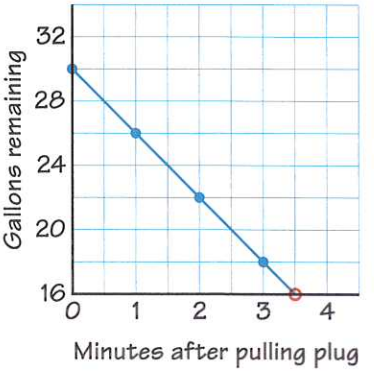
The answer is correct. After 3 minutes and 30 seconds, the tank will contain 16 gallons of water.

The reasoning behind the answer is explained clearly. To find my answer, I looked at the graph and saw that the tank will hold 16 gallons at $3\frac{1}{2}$ minutes. Because $\frac{1}{2}$ of a minute equals 30 seconds, I know that the tank will hold 16 gallons of water after 3 minutes and 30 seconds.

Partial credit solution

The table and graph are correct.

minutes	0	1	2	3	4
gallons	30	26	22	18	14



The answer is incorrect.

The tank will hold 16 gallons after 3 minutes and 5 seconds. The graph shows that there are 16 gallons after 3.5 minutes.

No credit solution

The table is correct, but there is no graph.

minutes	0	1	2	3	4
gallons	30	26	22	18	14

The answer is incorrect, and there is no explanation.

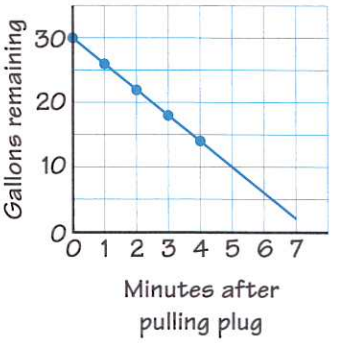
The tank will never have exactly 16 gallons in it.

Scoring is often based on how clearly you explain your reasoning.

Your turn now

1. Score one student's answer to the problem on page 692 as *full credit*, *partial credit*, or *no credit*. Explain your choice. If you choose *partial credit* or *no credit*, explain how to change the answer so that it earns *full credit*.

minutes	0	1	2	3	4
gallons	30	26	22	18	14



The tank will have 16 gallons after 3 minutes and 30 seconds. The table shows that the tank will have 16 gallons between 3 and 4 minutes. 16 is halfway between 14 and 18. So, the tank must have 16 gallons halfway between 3 and 4 minutes. The tank has 16 gallons at 3 minutes and 30 seconds.

UNIT 4
Chapters 11–13

PRACTICING Test-Taking Skills

Extended Response

1. The menu shows the prices of circular cheese pizzas with different diameters and the cost of each topping.

Diameter	Price	Cost of each topping
10 in.	\$4.25	\$.50
12 in.	\$6.25	\$.75
14 in.	\$7.75	\$.75
16 in.	\$8.75	\$1.00
18 in.	\$9.75	\$1.25
24 in.	\$13.00	\$1.75

The price of a 12 inch pizza can be modeled by the equation $p = 0.75t + 6.25$, where t is the number of toppings. Make a table of values that shows the prices of a 12 inch pizza with 1 to 6 toppings.

Graph the equation above using your table of values.

Write and graph an equation for the price of a 10 inch pizza. Which graph has a greater slope? Explain why.

2. The double stem-and-leaf plot shows the heights of 15 football players and 15 basketball players.

Heights of football players (in.) | **Heights of basketball players (in.)**

9	6	8
3 3 2 2 1 1 0	7	3 5 6 6
8 7 7 6 6 5 4		6 7 8 9
	8	0 0 1 4
		6 7

Key: 0 | 7 | 3 = 70 and 73

What is the height of the tallest football player?

How many basketball players are over 82 inches tall?

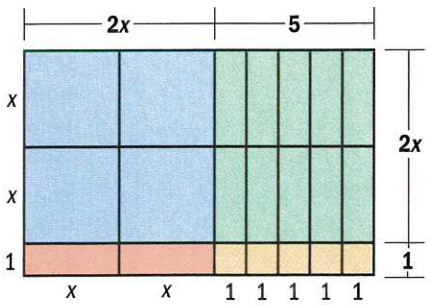
In general, are the football players or the basketball players taller? Explain how you used the data to draw a conclusion.

3. Players spin the two spinners shown and flip a coin. They earn a point for a 1, a point for blue, a point for tails, and a bonus point for getting all three in one turn.

In one turn, what is the probability of spinning a 1? spinning a blue? getting tails?

What is the probability of getting the bonus point? Explain your answer.

4. Write a polynomial expression for the area of each colored region of the figure shown.



Write a polynomial expression for the sum of these areas.

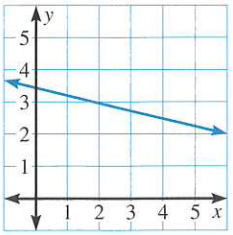
Write an expression for the area of the entire figure as a product of two binomials. Explain how you found your answer.

Multiple Choice

5. What is the slope of the line passing through the points $(4, -3)$ and $(-2, 5)$?

A. $-\frac{4}{3}$ **B.** -1 **C.** $\frac{3}{4}$ **D.** 1

6. Which equation is represented by the graph?



F. $y = \frac{1}{4}x + 3.5$ **G.** $y = -\frac{1}{4}x + 3.5$

H. $y = 4x + 3.5$ **I.** $y = -4x + 3.5$

7. Your locker combination has 3 numbers from 0 to 15. Each number must be different. How many locker combinations are possible?

A. 45 **B.** 48 **C.** 560 **D.** 3360

8. Find the difference.

$(5x^2 - 2x + 3) - (2x^2 - 2x - 3)$

F. $3x^2 + 6$ **G.** $3x^2 - 4x$

H. $3x^2 - 4x + 6$ **I.** $3x^2 + 4x - 6$

9. Find the product $(x - 6)(x + 3)$ and simplify.

A. $x^2 - 3x + 9$ **B.** $x^2 + 9x - 18$

C. $x^2 - 3x - 18$ **D.** $x^2 - 9x + 9$

Short Response

10. The table below shows the latitude and a typical minimum temperature in January for each of 6 U.S. cities.

City	Latitude (°N)	Minimum temperature in January (°F)
Anchorage	61	9
Miami	26	60
Helena	47	10
Reno	39	22
Buffalo	43	18
Memphis	35	31

Make a scatter plot of the data. What type of relationship does the scatter plot show between latitude and temperature? Explain.

11. The players of a baseball team hit the following numbers of home runs during one season: 6, 17, 12, 11, 6, 21, 5, 12, 9, 14, 24, 4, 25, 14, 18. Make a box-and-whisker plot of the data. Describe what the plot shows.

12. You roll two number cubes at the same time. What is the probability that the sum of the numbers showing on the two cubes is less than or equal to 5? What is the probability that the sum is greater than 5? Explain how you found your answers.

13. The function $S = 4\pi r^2$ gives the surface area of a sphere with radius r. Write this function in function notation. Graph the function. Estimate the radius of a sphere that has a surface area of 80 square centimeters. Use 3.14 for π. Explain your steps.

GO ON

UNIT 4
Chapters 11–13

PRACTICING **Test-Taking Skills**

Cumulative Practice for Chapters 11–13

Chapter 11

Multiple Choice **In Exercises 1–8, choose the letter of the correct answer.**

1. Which of the following relations is *not* a function? *(Lesson 11.1)*

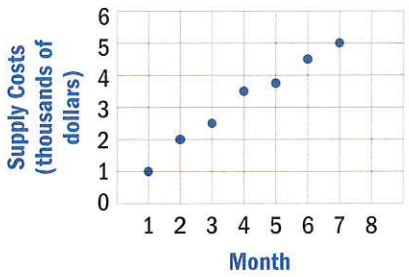
A. $(0, -3), (1, -1), (2, 1), (3, 3)$

B. $(-4, 0), (2, 1), (0, 2), (2, 3)$

C. $(0, 5), (2, 0), (4, -5), (-1, 3)$

D. $(6, 5), (0, -1), (3, 6), (-2, -1)$

2. The scatter plot shows the amount of money spent on supplies each month. What is the best prediction of the amount of money spent in the eighth month? *(Lesson 11.2)*



F. \$45 **G.** \$55 **H.** \$4500 **I.** \$5500

3. Which ordered pair is a solution of the equation $y = -2x + 6$? *(Lesson 11.3)*

A. $(6, -6)$ **B.** $(-4, 3)$

C. $(-8, -5)$ **D.** $(7, 7)$

4. Which equation is *not* linear? *(Lesson 11.4)*

F. $y = -2x + 4$ **G.** $3x + 4y = 8$

H. $y = x^3 - 8$ **I.** $4x = 8 - 2y$

5. What is the x-intercept of the graph of the equation $2x + 3y = 6$? *(Lesson 11.5)*

A. -3 **B.** -2 **C.** 2 **D.** 3

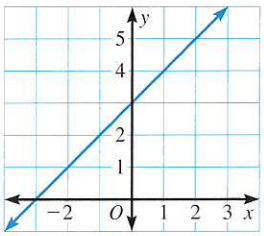
6. What is the slope of the line? *(Lesson 11.6)*

F. $-\frac{3}{2}$

G. 0

H. 1

I. $\frac{3}{2}$



7. A line passes through the points $(1, 2)$ and $(2, y)$ and has a slope of 0. What is the value of y? *(Lesson 11.6)*

A. -1 **B.** 2 **C.** 3 **D.** 5

8. Which ordered pair is a solution of $5x + 9y < 45$? *(Lesson 11.8)*

F. $(-3, 8)$ **G.** $(4, 6)$ **H.** $(6, -4)$ **I.** $(-2, 12)$

9. **Short Response** Write a function rule that relates x and y. *(Lesson 11.1)*

Input x	2	4	6	8
Output y	5	9	13	17

10. **Extended Response** For a party, you rent a karaoke machine for \$50. Food will cost \$6 per person. *(Lesson 11.7)*

a. Graph the equation $y = 6x + 50$ to see the possible total costs of the party.

b. How much will it cost to invite 11 people?

c. You want to spend no more than \$100. How many people can you invite?

Chapter 12

The stem-and-leaf plot shows the number of home runs for each player on a softball team. Use the plot for Exercises 11–12. *(Lesson 12.1)*

0	4 5 7 8	
1	2 4 5 6 6 8	
2	1 4 5	Key: 2 | 1 = 21

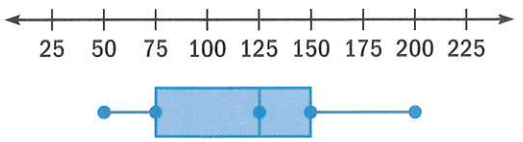
11. What is the greatest number of home runs hit by one player?

A. 4 **B.** 25 **C.** 28 **D.** 52

12. What is the median number of home runs?

F. 12 **G.** 15 **H.** 16 **I.** 21

The box-and-whisker plot shows stereo prices. Use the plot for Exercises 13–14. *(Lesson 12.2)*



13. What is the lowest stereo price?

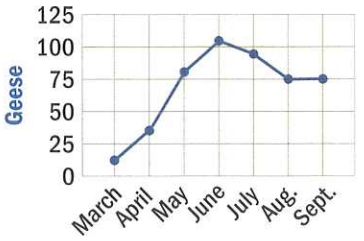
A. \$25 **B.** \$50 **C.** \$75 **D.** \$125

14. What is the median price of the stereos?

F. \$75 **G.** \$100 **H.** \$125 **I.** \$150

15. The line graph shows the average number of geese in one area over seven months. Which month had the greatest average number of geese? *(Lesson 12.3)*

A. April
B. May
C. June
D. July



16. You have 3 pairs of shoes, 6 pairs of pants, 4 shirts, and 3 belts. How many different combinations of shoes, pants, shirts, and belts can you make? *(Lesson 12.4)*

F. 16 **G.** 204 **H.** 216 **I.** 313

17. Four cyclists are racing. Trophies are awarded to the people who come in first place and second place. In how many different ways can the trophies be awarded? *(Lesson 12.5)*

A. 2 **B.** 4 **C.** 12 **D.** 24

18. There are 14 pints of rocky road ice cream and 12 pints of fudge almond in a freezer. If Joey chooses a pint at random, what is the probability that he will choose rocky road? *(Lesson 12.7)*

F. $\frac{6}{13}$ **G.** $\frac{7}{13}$ **H.** $\frac{6}{7}$ **I.** $\frac{7}{6}$

19. You roll a number cube once. What are the odds in favor of rolling a four? *(Lesson 12.7)*

A. $\frac{1}{6}$ **B.** $\frac{1}{5}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{3}$

20. A bag contains 16 red balls and 24 white balls. You draw a ball at random, replace it, and draw a second ball. What is the probability of drawing a red ball and then a white ball? *(Lesson 12.8)*

F. $\frac{1}{384}$ **G.** $\frac{1}{24}$ **H.** $\frac{1}{16}$ **I.** $\frac{6}{25}$

21. A bag contains the letters of the word *Mississippi*. You draw one letter at random, do not replace it, and then draw a second letter. What is the probability of drawing the letter *i* twice? *(Lesson 12.8)*

A. $\frac{12}{121}$ **B.** $\frac{6}{55}$ **C.** $\frac{16}{121}$ **D.** $\frac{8}{55}$

PRACTICING Test-Taking Skills

Cumulative Practice continued

22. Short Response A group of 10 people travel in 2 cars with 5 people in each car. How many different groups can go in one car? How many ways can the group split into 2 cars? Explain your answers. *(Lesson 12.6)*

23. Extended Response The points that you scored in each basketball game during the season are given below. Red numbers represent games won, and blue numbers represent games lost. *(Lesson 12.3)*

12, 10, 26, 22, 18, 17, 5, 20, 22, 18, 15, 8, 14, 19, 22, 23, 25, 8, 16, 20

a. Use an appropriate display to represent the data.

b. Explain your choice of display.

c. Identify a type of display that is not a good choice for the data. Explain why it is not a good choice.

Chapter 13

Multiple Choice In Exercises 24–30, choose the letter of the correct answer.

24. Simplify the polynomial. *(Lesson 13.1)*

$4(x^2 - 2x - 4) - 2x^2$

A. $-2x^2 + 8x + 16$ **B.** $2x^2 - 8x - 16$

C. $-2x^2 - 8x + 16$ **D.** $2x^2 + 8x + 16$

25. What is the value of $-3x^2 - 5x + 8$ for $x = 3$? *(Lesson 13.1)*

F. -34 **G.** -12 **H.** -6 **I.** 34

26. Find the sum $(2x - 6) + (3x - 9)$. *(Lesson 13.2)*

A. $5x - 15$ **B.** $-x - 3$

C. $x + 3$ **D.** $5x + 15$

27. Find the difference. *(Lesson 13.2)*

$(3x^4 - 2x^2 + 3) - (x^4 + 2x^3 + x^2)$

F. $2x^4 - 2x^3 - 3x^2 + 3$

G. $2x^4 - 2x^3 - x^2 + 3$

H. $2x^4 + 2x^3 - x^2 + 3$

I. $2x^4 - 4x^3 - x^2 + 3$

28. Simplify the expression $(3a^2)(4a + 1)$. *(Lesson 13.3)*

A. $7a^2 + 3a$ **B.** $7a^2 + 1$

C. $12a^3 + 3a^2$ **D.** $12a^3 + 3$

29. Simplify the expression $(3y^2)^3$. *(Lesson 13.3)*

F. $27y^5$ **G.** $3y^5$ **H.** $27y^6$ **I.** $3y^6$

30. Find the product and simplify. *(Lesson 13.4)*

$(3x + 3)(2x - 2)$

A. $6x^2 + 6$ **B.** $6x^2 - 6$

C. $6x^2 + 6x - 6$ **D.** $6x^2 - 6x + 6$

31. Short Response A walnut falls from a tree 75 feet off the ground. The polynomial $-16t^2 + 75$ gives the walnut's height, in feet, after t seconds of falling. Find its height after 1 second and 2 seconds. Estimate how many seconds the walnut falls before hitting the ground. *(Lesson 13.1)*

32. Extended Response Use your graphs to look for a pattern. *(Lesson 13.5)*

a. Graph each function in a coordinate plane.

$f(x) = 3x^2 - 5$
$f(x) = 3x^2 - 3$
$f(x) = 3x^2 - 1$

b. Describe the graph of the function $f(x) = 3x^2 - 100$.

End-of-Course Test

Algebra, Integers, and Equation Solving

1. The table shows the number of visits to your Web site for five days. Make a bar graph of the data.

Days	Mon	Tues	Wed	Thurs	Fri
Visits	12	18	21	25	28

Evaluate the expression.

2. $8 \cdot 4 + 3 \cdot 5$

3. $46 - 25 + 8 \div 2$

4. $(3 + 1)^3 \div 8 - 2$

5. $7a - 4$ when $a = 2$

6. $\frac{9}{2y - 5}$ when $y = 4$

7. x^4 when $x = 5$

Use a number line to order the integers from least to greatest.

8. $65, -13, 19, 61, -19, 34$

9. $878, 433, -602, 1074, -1222$

Find the sum, difference, product, or quotient.

10. $-61 + (-44)$

11. $-13 + 8 + (-6)$

12. $-49 - (-16)$

13. $85 - (-18) - 12$

14. $5(-8)$

15. $-9(-6)(-4)$

16. $\frac{-64}{8}$

17. $\frac{-55}{-11}$

18. In a local school committee election, 250 votes were cast for two candidates. Candidate A won the election by 10 votes. How many votes did Candidate A receive?

19. Plot and connect points $A(-10, 12)$, $B(10, 12)$, $C(10, -12)$, and $D(-10, -12)$. Identify the figure. Then find the perimeter and area of the figure.

Simplify the expression by combining like terms.

20. $6y + 5y$

21. $8a - 6b + a$

22. $6(x - 7) + 4x + 3$

23. $3(2x + y) - (8 + 5x)$

Solve the equation.

24. $49 = d + 12$

25. $b - 18 = 12$

26. $8 = \frac{56}{x}$

27. $15 = \frac{x}{5}$

28. $4b = -64$

29. $-6x = 72$

30. $5b + 9 = 18$

31. $-21 = 19 - 4x$

Solve the inequality. Then graph the solution.

32. $b + 6 \leq 12$

33. $6 > 8 + t$

34. $-\frac{1}{6}a < 24$

35. $8 \geq -\frac{1}{4}x$

Algebra and Rational Numbers

36. Write the following fractions in simplest form: $\frac{12}{26}, \frac{18}{54}, \frac{16}{82}, \frac{24}{84}$.

Find the GCF and LCM of the numbers.

37. 12, 16, 48 **38.** 16, 32, 64 **39.** 45, 90, 180 **40.** 10, 15, 50

Simplify. Write the expression using only positive exponents.

41. $y^6 \cdot y^4$ **42.** $x^{-7} \cdot x^5$ **43.** $\frac{x^{18}}{x^{11}}$ **44.** $y^8 \cdot y^{-5}$

Find the sum, difference, product, or quotient.

45. $\frac{13}{24} + \left(-\frac{7}{24}\right)$ **46.** $-2\frac{2}{5} - 2\frac{2}{5}$ **47.** $8\frac{1}{2} + 5\frac{1}{4}$ **48.** $11\frac{2}{9} - 14\frac{1}{2}$

49. $5\frac{1}{4} \cdot \left(-4\frac{1}{5}\right)$ **50.** $-3\frac{1}{3} \cdot \left(-8\frac{4}{5}\right)$ **51.** $1\frac{1}{2} \div \left(-3\frac{3}{8}\right)$ **52.** $5\frac{1}{9} \div 2\frac{4}{7}$

53. $7.9 + (-2.344)$ **54.** $-8.1 - (-4.06)$ **55.** $14.66 \cdot 2.1$ **56.** $4.844 \div (-0.56)$

57. Order the numbers from least to greatest: $8\frac{2}{3}$, 8.5, 13.35, $7\frac{2}{3}$, $8\frac{9}{20}$, $8.\overline{6}$, 7.7. Then find the mean, median, and mode(s) of the numbers.

Solve the equation or inequality.

58. $3(b - 9) = -39$ **59.** $-6 = 11a - 5a$ **60.** $3x = 9(x - 1)$ **61.** $6(y - 4) = 3(y + 9)$

62. $\frac{1}{6}n + \frac{2}{3}n = 1$ **63.** $5.9 + c = 3c - 2.1$ **64.** $5a + 8 \geq 12$ **65.** $-2(d + 4) < -4$

66. $\frac{2}{9} = \frac{10}{a}$ **67.** $\frac{x}{4} = \frac{21}{12}$ **68.** $\frac{45}{15} = \frac{y}{1}$ **69.** $\frac{12}{15} = \frac{p}{100}$

Write the number as a percent, as a decimal, and as a fraction.

70. 22.5% **71.** 0.37 **72.** $\frac{13}{20}$ **73.** 0.1%

74. You buy a stereo for \$74 plus 8% sales tax. What is your total cost?

75. What is 0.5% of 65? Five is what percent of 125?

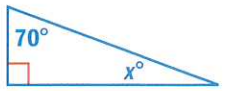
76. You draw a tile randomly from a bag that contains 12 B tiles, 6 C tiles, 8 D tiles, and 5 A tiles. What is the probability that you draw a D?

Geometry and Measurement

77. $\angle 1$ and $\angle 2$ are complementary, and $m\angle 1 = 74°$. Find $m\angle 2$.

Find the value of *x*. Then classify the figure.

78.



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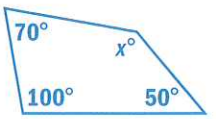

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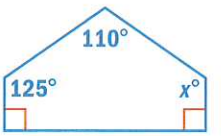


81. Draw $\triangle ABC$ in the coordinate plane with vertices $A(2, 5)$, $B(2, 2)$, and $C(6, 2)$. Draw $\triangle DEF$ congruent to $\triangle ABC$. Then translate $\triangle DEF$ using $(x, y) \longrightarrow (x - 4, y - 7)$. Identify the coordinates of $\triangle DEF$ and $\triangle D'E'F'$.

82. Find the following square roots: $-\sqrt{2}$, $\sqrt{121}$, $\sqrt{81}$, $-\sqrt{400}$

83. Is $\sqrt{\frac{4}{81}}$ a rational or irrational number? $\sqrt{18}$? Explain.

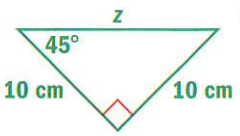
A right triangle has leg lengths *a* and *b* and a hypotenuse of length *c*. Find the unknown length. Then find the triangle's area and perimeter.

84. $a = 12$ cm, $b = 5$ cm, $c = ?$

85. $a = 12$ ft, $b = 16$ ft, $c = ?$

Find the value of each variable. Give exact answers.

86.



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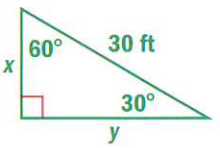

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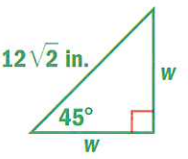
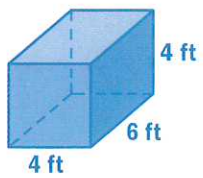


89. Find the area of a trapezoid with bases of 18 feet and 12 feet and a height of 5 feet.

90. What is the area of a circle with a radius of 5 feet? with a diameter of 12 centimeters? Use 3.14 for π.

Find the surface area and volume of the solid. Use 3.14 for π.

91.



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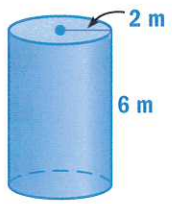

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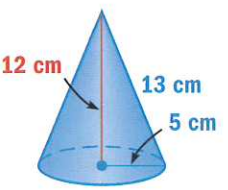

Advanced Algebra Topics

94. Make tables of values for the functions $y = 2x$ and $y = x - 2$. Use a domain of $-2, -1, 0, 1$, and 2. Identify the range of each function.

95. Make a scatter plot of (1, 4), (2, 8), (3, 12), and (4, 16). Describe the relationship between the variables. Then find the next ordered pair.

Graph the equation. Identify the intercepts of the graph.

96. $y = 2x - 12$ **97.** $y = 4 - 3x$ **98.** $y = -x + 17$ **99.** $y = -5x - 6$

100. $y = -\frac{1}{2}x + 5$ **101.** $y = -6x + 8$ **102.** $x + 9y = 18$ **103.** $y = 2x + 4$

104. Find the slope of the line passing through the points (5,8) and (0,5).

105. Rewrite $y + 12x = 6$ and $13x - 11y = 12$ in slope-intercept form. Then find the slope and the y-intercept of the graph of each equation.

106. Tell whether $(-8, -9)$ is a solution of the inequality $12x - 16y \geq 34$.

Make a stem-and-leaf plot and a box-and-whisker plot of the data.

107. 46, 49, 66, 51, 68, 83, 78, 64

108. \$68, \$63, \$51, \$43, \$53, \$38, \$60, \$66

109. In an election poll, 32% preferred Atkins, 47% preferred Liu, and 21% were undecided. Make a circle graph of the data.

110. You can choose from 3 sandwiches, 4 drinks, and 2 snacks. How many different sandwich-drink-snack groupings are possible?

Evaluate.

111. ${}_{18}P_2$ **112.** ${}_{14}P_3$ **113.** ${}_{9}C_6$ **114.** ${}_{8}C_5$

115. You roll a number cube. What are the odds of getting an odd number?

Simplify the polynomial and write it in standard form.

116. $4x^2 - 5x - 2x^2 + 7x$

117. $-3x^2 + 3x - x + 7$

Simplify the expression.

118. $(x^2 - 2x + 5) + (x^2 - 4)$ **119.** $(5y^2 - y - 1) - (y^2 + 4y)$ **120.** $(3x^2 - 4x + 2) + (x^2 + 6x - 5)$

121. $(6x)(8x^6)$ **122.** $(8y)^3$ **123.** $(x^6)^3$

124. $(n - 3)(n - 3)$ **125.** $(4x - 2)(8x - 3)$ **126.** $(12b - 4)(8b + 3)$

127. Graph $f(x) = x^2 + 6$. Use a table of values with $x = 2, 1, 0, -1$, and -2.