

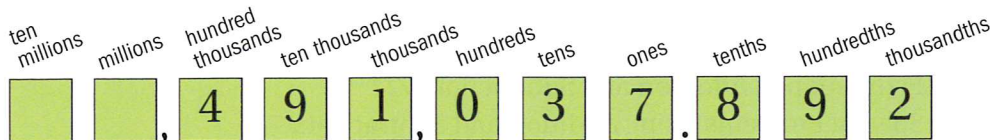
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Skills Review Handbook

Place Value

The **whole numbers** are the numbers 0, 1, 2, 3, A **digit** is any of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. The decimals are numbers such as 121.32, 25.6, and 3.456. For example, the decimal 4.5 has the digits 4 and 5. The place value of each digit in a whole number or a decimal depends on its position within the number. For example, in the number 491,037.892, the 8 has a value of 0.8 or 8×0.1 because it is in the tenths' place.



EXAMPLE Write the number 27,037.6 in expanded form.

$$27,037.6 = 20,000 + 7000 + 30 + 7 + 0.6$$

The zero in the hundreds' position is a placeholder.

$$= 2 \times 10,000 + 7 \times 1000 + 3 \times 10 + 7 \times 1 + 6 \times 0.1$$

EXAMPLE Write the number in standard form.

a. $4 \times 1000 + 5 \times 10 + 6 \times 0.1 + 8 \times 0.01$

b. Four million, sixty thousand, five and two thousandths

Solution

a. $4 \times 1000 + 5 \times 10 + 6 \times 0.1 + 8 \times 0.01 = 4000 + 50 + 0.6 + 0.08$
 $= 4050.68$

b. Write 4 in the millions' place, 6 in the ten thousands' place, 5 in the ones' place, and the 2 in the thousandths' place. Use zeros as placeholders for the other places. The answer is 4,060,005.002.

Practice

Write the number in expanded form.

1. 56,809

2. 3.075

3. 1002.003

4. 306.405

Write the number in standard form.

5. $5 \times 100,000 + 6 \times 10 + 9 \times 1 + 7 \times 0.001$ 6. Five million, ten and thirty-six thousandths

Rounding

To **round** a number means to approximate the number to a given place value. When rounding, look at the digit to the right of that place value. If the digit to the right is less than 5 (0, 1, 2, 3, or 4), round down. If the digit to the right is 5 or greater (5, 6, 7, 8, or 9), round up.

EXAMPLE Round the number to the place value of the red digit.

a. 6932

b. 45.674

Solution

- a. Because the 9 is in the hundreds' place, round 6932 to the nearest hundred. Notice that 6932 is between 6900 and 7000, so it will round to one of these two numbers.



Notice that 6932 is closer to 6900 than to 7000.

The digit to the right of the 9 in the hundreds' place is the 3 in the tens' place. Because 3 is less than 5, round down.

ANSWER 6932 rounded to the nearest hundred is 6900.

- b. Because 6 is in the tenths' place, round 45.674 to the nearest tenth. Notice that 45.674 is between 45.6 and 45.7, so it will round to one of these two numbers.



Notice that 45.674 is closer to 45.7 than to 45.6.

The digit to the right of the 6 in the tenths' place is the 7 in the hundredths' place. Because 7 is 5 or greater, round up.

ANSWER 45.674 rounded to the nearest tenth is 45.7.

Practice

Round the number to the place value of the red digit.

- | | | | | |
|-------------|------------|------------|--------------|---------------|
| 1. 1253 | 2. 57,309 | 3. 8.183 | 4. 32.76 | 5. 44,380 |
| 6. 12.535 | 7. 452.84 | 8. 998,543 | 9. 62.847 | 10. 640,796 |
| 11. 164.479 | 12. 1209.4 | 13. 52.961 | 14. 12,742.5 | 15. 3,501,652 |

Divisibility Tests

When two nonzero whole numbers are multiplied together, each number is a **factor** of the product. A number is **divisible** by another number if the second number is a factor of the first. For example, $2 \times 5 = 10$, so 2 and 5 are factors of 10, and 10 is divisible by both 2 and 5.

You can use the following tests to test a whole number for divisibility by 2, 3, 4, 5, 6, 8, 9, and 10.

- Divisible by 2: The last digit of the number is 0, 2, 4, 6, or 8.
- Divisible by 3: The sum of the digits of the number is divisible by 3.
- Divisible by 4: The last two digits of the number are divisible by 4.
- Divisible by 5: The last digit of the number is 0 or 5.
- Divisible by 6: The number is divisible by both 2 and 3.
- Divisible by 8: The last three digits of the number are divisible by 8.
- Divisible by 9: The sum of the digits of the number is divisible by 9.
- Divisible by 10: The last digit of the number is 0.

EXAMPLE Test the number for divisibility by 2, 3, 4, 5, 6, 8, 9, and 10.

a. 2736

b. 74,420

Solution

- a. The last digit of 2736 is 6, so it is divisible by 2 but not by 5 or 10. The sum of the digits is $2 + 7 + 3 + 6 = 18$, so it is divisible by 3 and 9. The last two digits, 36, are divisible by 4, so 2736 is divisible by 4. Because 2736 is divisible by both 2 and 3, it is divisible by 6. The last three digits, 736, are divisible by 8, so 2736 is divisible by 8.

ANSWER 2736 is divisible by 2, 3, 4, 6, 8, and 9.

- b. The last digit of 74,420 is 0, so it is divisible by 2, 5, and 10. The sum of the digits is $7 + 4 + 4 + 2 + 0 = 17$, so it is not divisible by 3 or 9. The last two digits, 20, are divisible by 4, so 74,420 is divisible by 4. Because 74,420 is divisible by 2, but not by 3, it is not divisible by 6. The last three digits, 420, are not divisible by 8, so 74,420 is not divisible by 8.

ANSWER 74,420 is divisible by 2, 4, 5, and 10.

Practice

Test the number for divisibility by 2, 3, 4, 5, 6, 8, 9, and 10.

- | | | | | |
|---------|---------|---------|-----------|------------|
| 1. 34 | 2. 84 | 3. 285 | 4. 560 | 5. 972 |
| 6. 4210 | 7. 2815 | 8. 6390 | 9. 88,004 | 10. 75,432 |

Mixed Numbers and Improper Fractions

A **fraction** is a number of the form $\frac{a}{b}$ ($b \neq 0$) where a is called the **numerator** and b is called the **denominator**. A number $1\frac{3}{5}$, read as “one and three fifths,” is a *mixed number*. A **mixed number** is the sum of a whole number part and a fraction part. An **improper fraction**, such as $\frac{21}{8}$, is any fraction in which the numerator is greater than or equal to the denominator.

EXAMPLE Write $3\frac{2}{5}$ as an improper fraction.

$$\begin{aligned} 3\frac{2}{5} &= \frac{15 + 2}{5} && \text{1 whole} = \frac{5}{5}, \text{ so 3 wholes} = \frac{3 \times 5}{5}, \text{ or } \frac{15}{5}. \\ &= \frac{17}{5} && \text{Add.} \end{aligned}$$

EXAMPLE Write $\frac{13}{4}$ as a mixed number.

1. Divide 13 by 4.

$$\begin{array}{r} 3R1 \\ 4 \overline{)13} \\ \underline{12} \\ 1 \end{array}$$

2. Write the mixed number. $3 + \frac{1}{4} = 3\frac{1}{4}$

Practice

Copy and complete the statement.

1. $7\frac{3}{5} = \frac{?}{5}$

2. $3\frac{1}{6} = \frac{?}{6}$

3. $\frac{23}{4} = 5\frac{?}{4}$

4. $\frac{17}{7} = 2\frac{?}{7}$

Write the mixed number as an improper fraction.

5. $3\frac{1}{2}$

6. $1\frac{5}{6}$

7. $4\frac{3}{8}$

8. $8\frac{5}{7}$

9. $10\frac{3}{4}$

Write the improper fraction as a mixed number.

10. $\frac{11}{4}$

11. $\frac{15}{2}$

12. $\frac{25}{6}$

13. $\frac{17}{3}$

14. $\frac{33}{8}$

Ratio and Rate

One way to compare numbers is to use a ratio. The **ratio** uses division to compare two numbers. You can write the ratio of a to b as $\frac{a}{b}$, as $a : b$, or as “ a to b .”

EXAMPLE

There are 15 boys and 17 girls in the band. Write the ratio of the number of boys to girls in the band in three ways.

$$\frac{\text{Number of boys}}{\text{Number of girls}} = \frac{15}{17} = 15 \text{ to } 17 = 15 : 17$$

A **rate** is a ratio of two quantities that have different units, such as $\frac{150 \text{ miles}}{3 \text{ hours}}$.

A **unit rate** is a rate with a denominator of 1 unit.

EXAMPLE

Write the rate $\frac{150 \text{ miles}}{3 \text{ hours}}$ as a unit rate.

$$\begin{array}{c} \div 3 \\ \curvearrowright \\ \frac{150 \text{ mi}}{3 \text{ h}} = \frac{50 \text{ mi}}{1 \text{ h}} \\ \curvearrowleft \\ \div 3 \end{array}$$

Divide 3 by 3 to get 1, so divide 150 by 3 also.

ANSWER The unit rate is 50 miles per hour.

Practice

The table shows the numbers of boys and girls in Mr. Smith's class and in Ms. Jung's class. Use the table to write the specified ratio.

- Boys in Mr. Smith's class to girls in Mr. Smith's class
- Boys in Mr. Smith's class to boys in Ms. Jung's class
- Girls in Ms. Jung's class to all girls

	Boys	Girls
Mr. Smith's class	13	12
Ms. Jung's class	17	11

Write the rate and unit rate.

- 8 feet in 2 seconds
- 333 miles in 6 hours
- 3 quarts for \$2.50
- \$24 for 8 pens
- 280 words in 5 minutes
- 8 inches in 6 days

Adding and Subtracting Decimals

To add and subtract decimals, start with the digits in the place on the right. Moving to the left, add or subtract the digits one place value at a time, regrouping as needed.

EXAMPLE Find the sum $0.157 + 0.663$.

- ① Add the thousandths.
Regroup 10 thousandths as 1 hundredth and 0 thousandths.

$$\begin{array}{r} 0.157 \\ + 0.663 \\ \hline 0 \end{array}$$

- ② Add the hundredths.
Regroup 12 hundredths as 1 tenth and 2 hundredths.

$$\begin{array}{r} 0.157 \\ + 0.663 \\ \hline 20 \end{array}$$

- ③ Add the tenths. Place the decimal point in the answer.

$$\begin{array}{r} 0.157 \\ + 0.663 \\ \hline 0.820 \end{array}$$

EXAMPLE Find the difference $30.7 - 3.8$.

- ① Start with the tenths.
There are not enough tenths in 30.7 to subtract 8 tenths.

$$\begin{array}{r} 30.7 \\ - 3.8 \\ \hline \end{array}$$

- ② Move to the ones.
There are no ones in 30.7, so regroup 1 ten as 9 ones and 10 tenths.

$$\begin{array}{r} 29.17 \\ - 3.8 \\ \hline \end{array}$$

- ③ Subtract. Place the decimal point in the answer.

$$\begin{array}{r} 29.17 \\ - 3.8 \\ \hline 26.9 \end{array}$$

✓ **Check** Because addition and subtraction are inverse operations, you can check your answer by adding: $26.9 + 3.8 = 30.7$.

Practice

Find the sum or difference.

- | | | | |
|----------------------|----------------------|----------------------|-------------------------|
| 1. $3.56 + 2.74$ | 2. $12.7 + 93.8$ | 3. $27.5 + 3.6$ | 4. $0.923 + 0.179$ |
| 5. $4.217 + 6.739$ | 6. $9.3 - 2.8$ | 7. $4.56 - 1.65$ | 8. $13.64 - 5.85$ |
| 9. $38.45 - 19.57$ | 10. $741.52 - 48.66$ | 11. $56.98 + 0.82$ | 12. $100.476 - 4.989$ |
| 13. $365.57 - 79.38$ | 14. $49.86 + 2.65$ | 15. $97.156 - 9.092$ | 16. $232.543 - 209.692$ |

Adding and Subtracting Fractions

To add two fractions with a common denominator, write the sum of the numerators over the denominator.

Numbers $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

Algebra $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} (c \neq 0)$

EXAMPLE Find the sum $\frac{4}{7} + \frac{6}{7}$.

$$\frac{4}{7} + \frac{6}{7} = \frac{4+6}{7}$$

Write sum of numerators over denominator.

$$= \frac{10}{7}$$

Add.

$$= 1\frac{3}{7}$$

Write the improper fraction as a mixed number.

To subtract two fractions with a common denominator, write the difference of the numerators over the denominator.

Numbers $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$

Algebra $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c} (c \neq 0)$

EXAMPLE Find the difference $\frac{10}{11} - \frac{4}{11}$.

$$\frac{10}{11} - \frac{4}{11} = \frac{10-4}{11}$$

Write difference of numerators over denominator.

$$= \frac{6}{11}$$

Subtract.

Practice

Find the sum or difference.

1. $\frac{1}{3} + \frac{1}{3}$

2. $\frac{8}{9} + \frac{5}{9}$

3. $\frac{6}{7} - \frac{3}{7}$

4. $\frac{11}{12} - \frac{4}{12}$

5. $\frac{1}{8} + \frac{7}{8}$

6. $\frac{8}{11} + \frac{7}{11}$

7. $\frac{13}{15} - \frac{2}{15}$

8. $\frac{5}{6} - \frac{4}{6}$

9. $\frac{1}{9} + \frac{1}{9}$

10. $\frac{10}{11} - \frac{2}{11}$

11. $\frac{11}{12} + \frac{8}{12}$

12. $\frac{9}{10} - \frac{6}{10}$

13. $\frac{5}{9} - \frac{4}{9}$

14. $\frac{9}{16} + \frac{12}{16}$

15. $\frac{11}{14} - \frac{2}{14}$

16. $\frac{8}{15} - \frac{8}{15}$

17. $\frac{5}{12} + \frac{2}{12}$

18. $\frac{8}{10} + \frac{1}{10}$

19. $\frac{6}{7} + \frac{5}{7}$

20. $\frac{5}{8} - \frac{2}{8}$

Estimation in Addition and Subtraction

To **estimate** a solution means to find an approximate answer. When numbers being added have about the same value, you can use *clustering* to estimate their sum. Another way to estimate is to add the digits in the greatest place, then round the remaining parts of the numbers and add. Finally, add the sums together.

EXAMPLE Estimate the sum $3836 + 4235 + 3982$.

3836	4000	The numbers all cluster around the value 4000.
4235	4000	
+ 3982	+ 4000	
	12,000	

ANSWER The sum $3836 + 4235 + 3982$ is *about* 12,000.

To estimate a difference, first subtract the digits in the greatest place. Then round the remaining parts of the numbers and subtract the lesser number from the greater number. Finally, combine the two differences using addition or subtraction as shown below.

EXAMPLE Estimate the difference $68,453 - 32,792$.

- | | | |
|--|---------------------------|------------|
| ① First subtract the digits in the ten thousands' place. | $68,453$ | $60,000$ |
| | $- 32,792$ | $- 30,000$ |
| ② Then round the remaining digits to the nearest thousand. Subtract the lesser number from the greater number. | $8,000$ | $30,000$ |
| | $- 3,000$ | |
| | $5,000$ | |
| ③ Because the greater remaining number was originally on the top, you <i>add</i> the differences. | $30,000 + 5,000 = 35,000$ | |

Note that if the greater remaining number had originally been on the *bottom*, you would *subtract* the differences.

Practice

Estimate the sum or difference.

- | | | |
|----------------------|-------------------------|-------------------------|
| 1. $935 + 887 + 912$ | 2. $4967 + 4802 + 5218$ | 3. $5971 + 6032 + 7865$ |
| 4. $8891 - 4932$ | 5. $4373 - 2158$ | 6. $449,739 - 285,921$ |

Solving Problems Using Addition and Subtraction

You can use the following guidelines to tell whether to use addition or subtraction to solve a word problem.

- Use addition when you need to combine, join, or find a total.
- Use subtraction when you need to separate, compare, take away, find how many are left, or find how many more are needed.

EXAMPLE You have 36 stamps in your stamp collection. You want to collect 18 more stamps. How many stamps will you have in all?

You need to combine, so you need to add.

$$36 + 18 = 54$$

ANSWER You will have 54 stamps in your stamp collection.

EXAMPLE Your total bill for lunch is \$4.78. You pay with a \$5 bill. How much change do you receive?

You need to take away, so you need to subtract.

$$\$5.00 - \$4.78 = \$0.22$$

ANSWER You receive \$.22 in change.

Practice

1. You spend \$48 for a coat and \$45 for a pair of shoes. How much do you spend in all?
2. You bought a box of 96 pencils. You gave 28 of the pencils to your friend. How many pencils do you have left?
3. You have \$18. You buy a video for \$15.99. How much money do you have left?
4. You have 24 country CDs and 18 pop CDs. How many country and pop CDs do you have in all?
5. You have 900 minutes a month on your cell phone plan. You have used 652 minutes so far this month. How many minutes do you have left?
6. You have \$149. You make \$24 babysitting. How much money do you have?

Multiplying Fractions

To multiply a fraction by a whole number, multiply the numerator of the fraction by the whole number and write the product over the denominator of the fraction. Simplify if possible.

EXAMPLE Find the product.

a. $3 \times \frac{2}{7} = \frac{3 \times 2}{7}$

Write the product of the whole number and the numerator over the denominator.

$$= \frac{6}{7}$$

Multiply.

b. $\frac{3}{8} \times 5 = \frac{3 \times 5}{8}$

Write the product of the whole number and the numerator over the denominator.

$$= \frac{15}{8}, \text{ or } 1\frac{7}{8}$$

Multiply. Then write as a mixed number.

To multiply two fractions, write the product of the numerators over the product of the denominators. Simplify if possible.

$$\text{product of fractions} = \frac{\text{product of numerators}}{\text{product of denominators}}$$

EXAMPLE Find the product.

$$\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3}$$

Use rule for multiplying fractions.

$$= \frac{8}{15}$$

Multiply.

Practice

Find the product. Simplify if possible.

1. $6 \times \frac{2}{15}$

2. $2 \times \frac{6}{11}$

3. $4 \times \frac{5}{9}$

4. $8 \times \frac{5}{9}$

5. $\frac{3}{4} \times 7$

6. $\frac{4}{7} \times 5$

7. $\frac{6}{7} \times 3$

8. $\frac{3}{7} \times \frac{6}{11}$

9. $\frac{2}{3} \times \frac{4}{5}$

10. $\frac{3}{4} \times \frac{1}{7}$

11. $\frac{1}{8} \times \frac{3}{5}$

12. $\frac{7}{9} \times \frac{5}{8}$

13. $\frac{5}{9} \times \frac{2}{3}$

14. $\frac{3}{8} \times \frac{4}{5}$

15. $\frac{5}{12} \times \frac{5}{6}$

Multiplication of a Decimal by a Whole Number

To multiply a decimal by a whole number, multiply the entire first number (ignoring the decimal point) by the digit in each place value of the second number to get partial products. Add the partial products. Then place the decimal point in the answer, showing the same number of decimal places as in the decimal.

EXAMPLE Find the product 31.5×206 .

- ① Multiply 31.5 by the ones' digit in 206. Ignore the decimal point.

$$\begin{array}{r} 3 \\ 31.5 \\ \times 206 \\ \hline 1890 \end{array}$$

- ② Skip the 0 in the tens' place, and multiply by the hundreds' digit. Start the partial product in the hundreds' place.

$$\begin{array}{r} 1 \\ 31.5 \\ \times 206 \\ \hline 1890 \\ 630 \end{array}$$

- ③ Add the partial products. The decimal has one decimal place, so show one decimal place in the answer.

$$\begin{array}{r} 31.5 \\ \times 206 \\ \hline 1890 \\ 630 \\ \hline 6489.0 \end{array}$$

Practice

Find the product.

1. $\begin{array}{r} 2.3 \\ \times 98 \\ \hline \end{array}$

2. $\begin{array}{r} 0.62 \\ \times 46 \\ \hline \end{array}$

3. $\begin{array}{r} 85 \\ \times 7.9 \\ \hline \end{array}$

4. $\begin{array}{r} 0.56 \\ \times 63 \\ \hline \end{array}$

5. $\begin{array}{r} 2.08 \\ \times 14 \\ \hline \end{array}$

6. $\begin{array}{r} 6.52 \\ \times 36 \\ \hline \end{array}$

7. $\begin{array}{r} 7.24 \\ \times 89 \\ \hline \end{array}$

8. $\begin{array}{r} 8.35 \\ \times 16 \\ \hline \end{array}$

9. $\begin{array}{r} 77.6 \\ \times 22 \\ \hline \end{array}$

10. $\begin{array}{r} 3.45 \\ \times 105 \\ \hline \end{array}$

11. $\begin{array}{r} 453 \\ \times 41.2 \\ \hline \end{array}$

12. $\begin{array}{r} 614 \\ \times 6.71 \\ \hline \end{array}$

13. $\begin{array}{r} 32.6 \\ \times 463 \\ \hline \end{array}$

14. $\begin{array}{r} 71.8 \\ \times 934 \\ \hline \end{array}$

15. $\begin{array}{r} 90.5 \\ \times 407 \\ \hline \end{array}$

16. $\begin{array}{r} 15.36 \\ \times 123 \\ \hline \end{array}$

17. $\begin{array}{r} 3.442 \\ \times 276 \\ \hline \end{array}$

18. $\begin{array}{r} 93.08 \\ \times 306 \\ \hline \end{array}$

19. $\begin{array}{r} 5.436 \\ \times 682 \\ \hline \end{array}$

20. $\begin{array}{r} 60.97 \\ \times 708 \\ \hline \end{array}$

21. $\begin{array}{r} 142.82 \\ \times 35 \\ \hline \end{array}$

22. $\begin{array}{r} 25.987 \\ \times 76 \\ \hline \end{array}$

23. $\begin{array}{r} 32.903 \\ \times 55 \\ \hline \end{array}$

24. $\begin{array}{r} 243.72 \\ \times 38 \\ \hline \end{array}$

25. $\begin{array}{r} 75.032 \\ \times 73 \\ \hline \end{array}$

26. $\begin{array}{r} 380.07 \\ \times 114 \\ \hline \end{array}$

27. $\begin{array}{r} 508.25 \\ \times 237 \\ \hline \end{array}$

28. $\begin{array}{r} 15.456 \\ \times 591 \\ \hline \end{array}$

29. $\begin{array}{r} 36.902 \\ \times 205 \\ \hline \end{array}$

30. $\begin{array}{r} 8257.6 \\ \times 459 \\ \hline \end{array}$

Dividing Decimals

In a division problem, the number being divided is called the **dividend** and the number it is being divided by is called the **divisor**. The result of the division is called the **quotient**. To **divide** two numbers, you start with the leftmost digit of the dividend and move to the right. Before you start dividing decimals, place the decimal point in the quotient.

EXAMPLE Find the quotient $5.2 \div 8$.

- (1) Place the decimal point in the quotient directly above the decimal point in the dividend. Then divide as with whole numbers. Because 8 is greater than 5, place a zero above the 5.

$$\text{divisor} \rightarrow \overset{0.}{8} \overline{)5.2} \leftarrow \text{dividend}$$

- (2) Because $8 \times 6 = 48$, estimate that 8 divides 52 about 6 times. Multiply 6 and 8. Then subtract 48 from 52. Be sure the difference is less than the divisor: $4 < 8$.

$$\begin{array}{r} 0.6 \\ 8 \overline{)5.2} \\ - 48 \\ \hline 4 \end{array}$$

- (3) Add **zero** as a placeholder. Bring down the zero. Divide 40 by 8 to get 5. Multiply 5 and 8. Subtract 40 from 40. The remainder is zero.

$$\begin{array}{r} 0.65 \\ 8 \overline{)5.20} \\ - 48 \\ \hline 40 \\ - 40 \\ \hline 0 \end{array}$$

EXAMPLE Find the quotient $12 \div 2.8$.

- (1) To multiply the divisor and the dividend by 10, move both decimal points 1 place to the right. Then divide as with whole numbers.

$$\text{divisor} \rightarrow 28 \overline{)120} \leftarrow \text{dividend}$$

- (2) Because $28 \times 4 = 112$, estimate that 28 divides 120 about 4 times. Multiply 4 and 28. Then subtract 112 from 120.

$$\begin{array}{r} 4 \\ 28 \overline{)120} \\ - 112 \\ \hline 8 \end{array}$$

- (3) Be sure the difference is less than the divisor: $8 < 28$. So $12 \div 2.8$ is equal to $4 \frac{8}{28}$, or $4 \frac{2}{7}$.

$$\begin{array}{r} 4R8 \\ 28 \overline{)120} \\ - 112 \\ \hline 8 \end{array}$$

Practice

Find the quotient.

1. $2.7 \div 6$

2. $3.8 \div 4$

3. $6.8 \div 8$

4. $46.9 \div 7$

5. $13.71 \div 3$

6. $15 \div 2.5$

7. $8 \div 1.3$

8. $32 \div 5.46$

9. $63 \div 7.12$

10. $75 \div 6.357$

Estimation in Multiplication and Division

One way to estimate a product or a quotient is to find a range for the product or quotient by finding a low estimate and a high estimate. A low estimate and a high estimate can be found by using *compatible numbers*, which are numbers that make a calculation easier.

EXAMPLE Find a low and high estimate for the product 56×35 using compatible numbers.

For a low estimate, round both factors *down*. $50 \times 30 = 1500$

For a high estimate, round both factors *up*. $60 \times 40 = 2400$

ANSWER The product 56×35 is between 1500 and 2400.

EXAMPLE Find a low and high estimate for the quotient $23,400 \div 45$ using compatible numbers.

When the divisor has more than one digit, round it .

For a *low* estimate, round the divisor *up* and choose a compatible dividend that is *lower* than the original dividend.

$$\begin{array}{r} 400 \\ 50 \overline{)20,000} \end{array}$$

For a *high* estimate, round the divisor *down* and choose a compatible dividend that is *higher* than the original dividend.

$$\begin{array}{r} 600 \\ 40 \overline{)24,000} \end{array}$$

ANSWER The quotient $23,400 \div 45$ is between 400 and 600.

Practice

Find a low and high estimate for the product or quotient using compatible numbers.

- | | | | |
|---------------------|----------------------|----------------------|----------------------|
| 1. 43×16 | 2. 359×28 | 3. 852×53 | 4. 734×76 |
| 5. $225 \div 6$ | 6. $2795 \div 7$ | 7. $17,934 \div 77$ | 8. $41,042 \div 92$ |
| 9. 326×48 | 10. 612×273 | 11. 745×158 | 12. 905×657 |
| 13. 625×28 | 14. 809×97 | 15. $742 \div 8$ | 16. $231 \div 38$ |
| 17. $5421 \div 7$ | 18. $4972 \div 18$ | 19. $1583 \div 82$ | 20. $43,789 \div 64$ |

Solving Problems Using Multiplication and Division

You can use the following guidelines to tell whether to use multiplication or division to solve a word problem.

- Use multiplication when you need to find the total number of objects that are in groups of equal size or to find a fractional part of another number.
- Use division when you need to find the number of equal groups or the number in each equal group.

EXAMPLE You baked 48 muffins. You give $\frac{1}{3}$ of them to your friend.
How many muffins did you give to your friend?

You need to find the fractional part of another number, so you need to multiply.

$$48 \cdot \frac{1}{3} = 16$$

ANSWER You gave 16 muffins to your friend.

EXAMPLE You bought 4 cans of soup for a total of \$3.56. How much did you pay for each can of soup?

You need to find the amount in each equal group, so you need to divide.

$$3.56 \div 4 = 0.89$$

ANSWER You paid \$.89 for each can of soup.

Practice

1. You bought 6 notebooks. Each notebook cost \$1.58. How much did you pay for all of the notebooks?
2. You have 92 baseball cards. You give $\frac{1}{4}$ of your cards to your friend. How many cards did you give your friend?
3. You have 12 flats of flowers. If each flat contains 48 flowers, how many flowers do you have?
4. You paid \$22.95 for the plates for your party. If you bought 9 packages of plates, how much did you pay for each package?

Points, Lines, and Planes

In geometry, a **point** is usually labeled with an uppercase letter, such as *A* or *B*. Points are used to name *lines*, *rays*, and *segments*. A **plane** is a flat surface that extends without end in all directions. You can represent a plane by a figure that looks like a floor or a wall.

Words	Diagram	Symbols
A line extends without end in two <i>opposite</i> directions.		\overleftrightarrow{XY} or \overleftrightarrow{YX}
A ray has one endpoint and extends without end in <i>one</i> direction.		\overrightarrow{XY}
A segment has two endpoints.		\overline{XY} or \overline{YX}

EXAMPLE Identify and name the *line*, *ray*, or *segment*.



Solution

- The figure is a segment that can be named \overline{AB} .
- The figure is a line that can be named \overleftrightarrow{MN} .
- The figure is a ray that can be named \overrightarrow{PQ} .

Practice

Match the name with the correct figure.

1. \overline{CD}

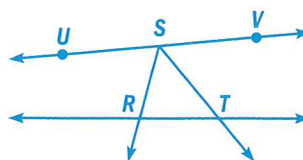
2. \overleftrightarrow{CD}

3. \overrightarrow{CD}



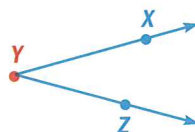
In Exercises 4–7, use the diagram.

- Name three points.
- Name two rays.
- Name two lines.
- Name a segment that has *S* as an endpoint.



Angles

An **angle** is formed by two rays with the same endpoint. The endpoint is called the **vertex**. The symbol \angle is used to represent an angle.



EXAMPLE Name the angle above in three ways.

Name the angle by its vertex alone: $\angle Y$

Name the angle by its vertex and two points, with the vertex as the middle point: $\angle XYZ$

Name the angle by its vertex and two points, but reverse the order of the points: $\angle ZYX$

Practice

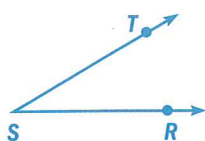
Match the name with the correct angle.

1. $\angle ABC$

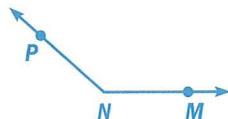
2. $\angle RST$

3. $\angle MNP$

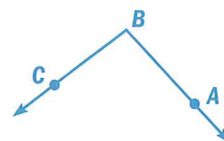
A.



B.

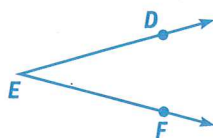


C.

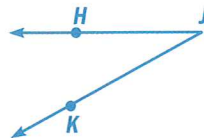


Name the angle in three ways.

4.



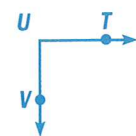
5.



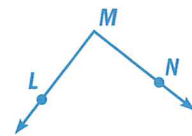
6.



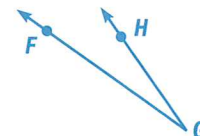
7.



8.



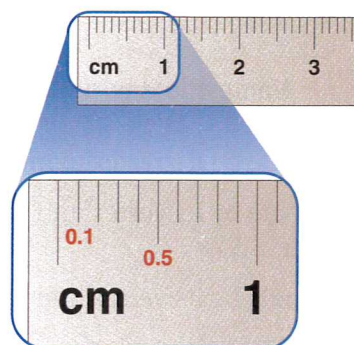
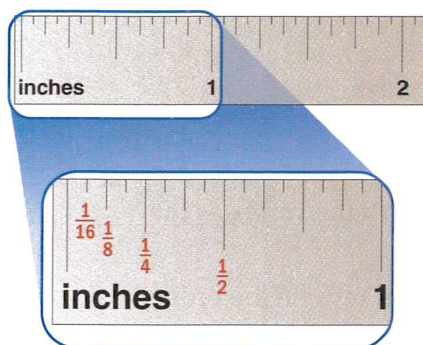
9.



10. Draw a triangle with vertices P , Q , and R . Name each angle of the triangle in three ways.

Using a Ruler

An **inch ruler** has markings for inches, halves of an inch, fourths of an inch, eighths of an inch, and sixteenths of an inch. As the lengths get shorter, so do the markings.



A **centimeter ruler** has markings for centimeters, halves of a centimeter, and tenths of a centimeter (also called *millimeters*). Like an inch ruler, as the lengths get shorter, so do the markings.

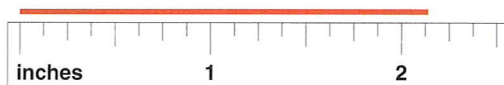
EXAMPLE Use a ruler to draw a segment with the given length.

a. $2\frac{1}{8}$ inches

b. 3.8 centimeters

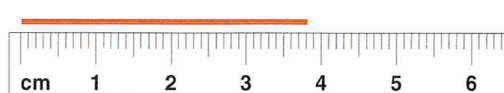
Solution

a. Start at the leftmost mark on the ruler.



Draw a segment so that the other end is at the first $\frac{1}{8}$ in. mark after 2.

b. Start at the leftmost mark on the ruler.



Draw a segment so that the other end is at the 3.8 cm mark.

Practice

Use a ruler to draw a segment with the given length.

1. $\frac{7}{16}$ inch

2. $4\frac{5}{8}$ inches

3. 4.3 centimeters

4. 2.7 centimeters

5. $2\frac{5}{16}$ inches

6. 6.5 centimeters

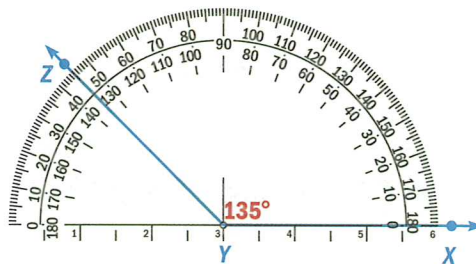
7. 2.9 centimeters

8. $1\frac{1}{4}$ inches

Using a Protractor

A **protractor** is a tool you can use to draw and measure angles. A unit of measure for angles is the **degree** ($^{\circ}$). To measure an angle, place the center of the protractor on the vertex of the angle and line up one ray with the 0° line. Then read the measure where the other ray crosses the protractor.

The measure of $\angle XYZ$ is 135° . You can write this as $m\angle XYZ = 135^{\circ}$.

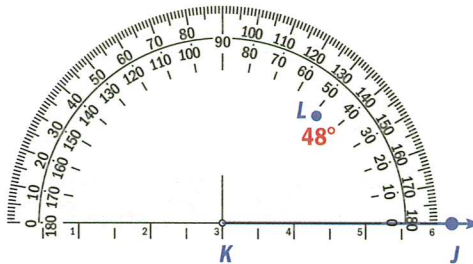


EXAMPLE Use a protractor to draw an angle that has a measure of 48° .

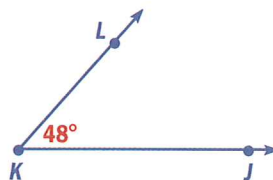
- 1 Draw and label a ray.



- 2 Place the center of the protractor at the endpoint of the ray. Line up the ray with the 0° line. Then draw and label a point at the 48° mark on the inner scale.

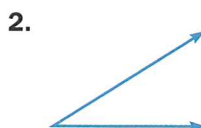


- 3 Remove the protractor and draw \overrightarrow{KL} to complete the angle.



Practice

Use a protractor to measure the angle.



4. Use a protractor to draw angles measuring 46° , 125° , and 73° .

Using a Compass

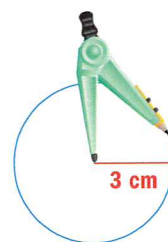
A **compass** is an instrument used to draw circles. A **straightedge** is any object that can be used to draw a segment.

EXAMPLE Use a compass to draw a circle with radius 3 cm.

Recall that the *radius* of a circle is the distance between the center of the circle and any point on the circle.

Use a metric ruler to open the compass so that the distance between the point and the pencil is 3 cm.

Place the point on a piece of paper and rotate the pencil around the point to draw the circle.



EXAMPLE Use a straightedge and a compass to draw a segment whose length is the sum of \overline{MN} and \overline{PQ} .

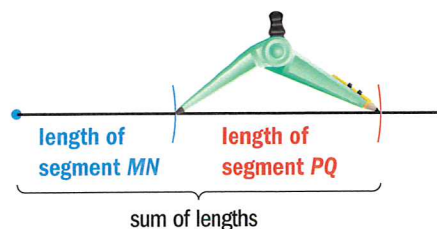


Solution

Use a straightedge to draw a segment longer than both given segments.

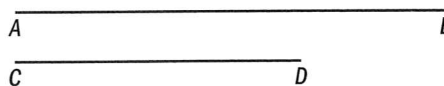
Open your compass to measure \overline{MN} . Using this compass setting, place the point at the left end of your segment and make a mark that crosses your segment.

Then open your compass to measure \overline{PQ} . Using this compass setting, place the point at the first mark you made on your segment and make another mark that crosses your segment.



Practice

1. Use a compass to draw a circle with radius 4 cm.
2. Use a straightedge and a compass to draw a segment whose length is the *sum* of the lengths of the two given segments.
3. Use a straightedge and a compass to draw a segment whose length is the *difference* of the lengths of the two given segments in Exercise 2.



Reading and Making Line Plots

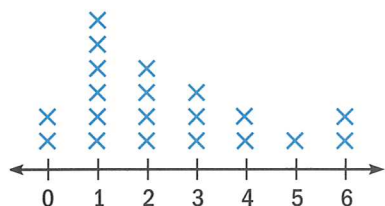
A **line plot** uses a number line to show how often data values occur.

EXAMPLE You surveyed 20 of your friends and asked them how many pets they have. Their responses were:
6, 2, 3, 1, 5, 0, 2, 4, 1, 1, 6, 2, 0, 3, 1, 4, 3, 2, 1, 1.

- a. Make a line plot of the data. b. What was the most frequent response?

Solution

a.



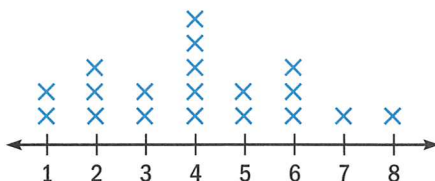
- b. The greatest number of ×s is above 1, so 1 was the most frequent response.

Practice

Make a line plot of the data.

- In a survey, 15 people were asked how many TVs they own. Their responses were: 1, 2, 1, 4, 3, 2, 1, 5, 1, 2, 3, 1, 2, 1, 2.
- In a survey, 18 people were asked how many times they eat out each week. Their responses were: 2, 4, 1, 3, 5, 6, 3, 7, 2, 1, 4, 8, 5, 4, 3, 4, 1, 2.

Use the line plot below. It shows the results of a questionnaire asking people how many hours they exercise each week.



- How many people completed the questionnaire?
- How many more people exercise 4 hours each week than exercise 6 hours each week?
- How many people exercise less than 3 hours each week?

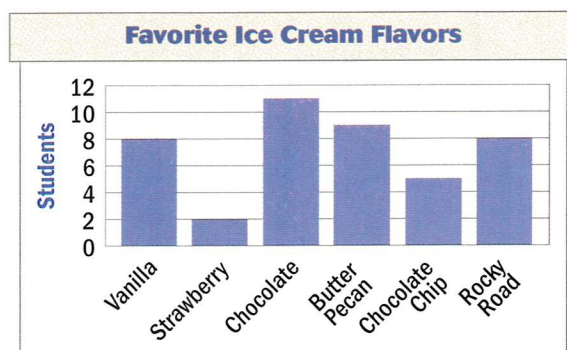
Reading and Making Bar Graphs

Data are numbers or facts. A **bar graph** is one way to display data. A bar graph uses bars to show how quantities in categories compare.

EXAMPLE The bar graph shows the results of a survey on favorite flavors of ice cream. Which flavor was chosen the most? Which flavor was chosen the least?

The longest bar on the graph represents the 11 people who chose chocolate. So, chocolate was chosen the most.

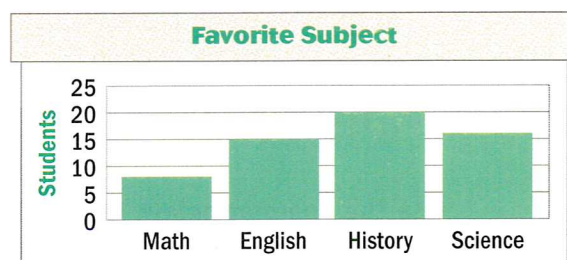
The shortest bar represents the 2 students who chose strawberry. So, strawberry was chosen the least.



To make a bar graph, choose a title and a scale. Draw and label the axes. Then draw bars to represent the data given.

EXAMPLE Draw a bar graph for the data given.

Subject	Number of Students
Math	8
English	15
History	20
Science	16



Practice

In Exercises 1–3, use the bar graph of ice cream flavors above.

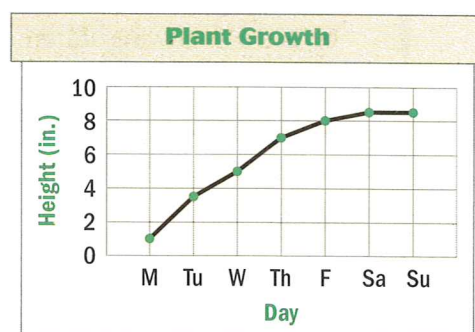
- How many students chose butter pecan as a favorite ice cream flavor?
- How many students chose vanilla as a favorite ice cream flavor?
- Which two flavors were chosen by the same number of students?
- Suppose 8 more students took the survey shown in the second example. Draw a new bar graph if 3 of the students chose math, 4 chose history, and 1 chose science.

Reading and Making Line Graphs

Another way to display data is to use a *line graph*. A **line graph** uses line segments to show how a quantity changes over time.

EXAMPLE The line graph shows plant growth data collected by students every day for 7 days. The greatest increase in growth occurred between what two days? What was the amount of the increase?

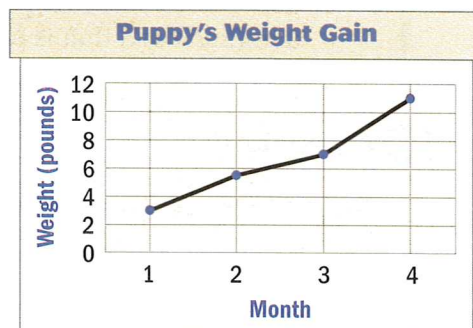
The steepest segment in the line graph is from Monday to Tuesday. The students recorded a height of 1 inch on Monday and a height of 3.5 inches on Tuesday, for an increase of 2.5 inches.



To make a line graph, choose a title and scales. Draw and label the axes. Then plot and connect points to represent the data given.

EXAMPLE Draw a line graph for the data given.

Month	Weight of Puppy (pounds)
1	3
2	5.5
3	7
4	11



Practice

In Exercises 1 and 2, use the plant growth line graph above.

- Between which two days was the growth 1 inch?
- Between which two days did the height remain the same?
- Suppose in month 5 the puppy in the second example weighed 15 pounds. Copy the graph and add this data to it.

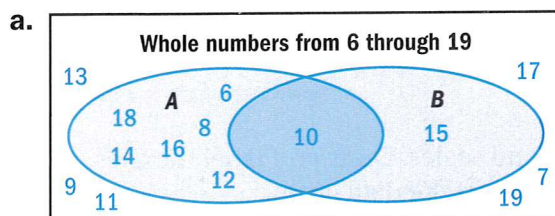
Venn Diagrams and Logical Reasoning

A **Venn diagram** uses shapes to show how sets are related.

EXAMPLE Draw and use a Venn diagram.

- Draw a Venn diagram of the whole numbers from 6 through 19 where set A consists of even numbers and set B consists of multiples of 5.
- Is the following statement *true* or *false*? Explain.
No even whole number from 6 through 19 is a multiple of 5.
- Is the following statement *always*, *sometimes*, or *never* true? Explain. *A multiple of 5 from 6 through 19 is even.*

Solution



- False. 10 is an even whole number that is a multiple of 5.
- Sometimes. It is true that 10 is a multiple of 5 that is even, but 15 is a multiple of 5 that is odd.

Practice

Draw a Venn diagram of the sets described.

- Of the whole numbers less than 12, set A consists of numbers that are greater than 8 and set B consists of odd numbers.
- Of the whole numbers less than 10, set C consists of multiples of 3 and set D consists of even numbers.

Use the Venn diagrams you drew in Exercises 1 and 2 to answer the question. Explain your reasoning.

- Is the following statement *true* or *false*?
There is only one odd number greater than 8 and less than 12.
- Is the following statement *always*, *sometimes*, or *never* true?
A whole number less than 10 is both a multiple of 3 and even.